TN Department of Education

Grades 1-2 Numeracy

Participant Manual

Tennessee Department of Education | 2016 Regional Educator Summit

Welcome, Participants!

We're excited to welcome you to this summer's **Regional Educator Summit**. We're impressed by your desire for professional learning and growth, and we hope you find this course productive and inspiring. As you engage in this training content over the next two days, we hope you make many connections to your own classroom practice. We look forward to hearing about the ways you implement this course content in the upcoming school year!

We are also proud to share that the content of this training was developed **by Tennessee educators, for Tennessee educators**. We believe it's important for professional development to be informed by current educators, who work in schools with students daily.

In particular, we'd like to thank the following educators who contributed to the creation and review of this content:

Content Developers

- Season Epps - Instructional Coach, Rutherford County Schools
- Dana Westveer - Kindergarten Teacher, Metro Nashville Public Schools
- Susan Armstrong - East Tennessee CORE Office

Content Reviewers

- Allison Painter - Putnam County
- Christy Grubb - Anderson County
- Kimberly Osborne - Murfreesboro City
- Heather Drinnon - Hamblen County

Module 4: Addition and Subtraction

Introduction

Introduction

Supporting PreK-2 Educators

Beginning in 2012, the Tennessee Department of Education offered summer trainings with a focus on both the early foundational skills in math and on teaching mathematics content standards and practice standards through high-level instructional tasks. The materials for the previous trainings can be accessed and downloaded from http://tncore.org/training.aspx until July 31, 2016. After that date, the materials will be located at http://edutoolbox.org/.

Our Focus in 2016

The focus of the 2016 Regional Educator Summit for early numeracy is to continue standards-driven learning that will support all students in achieving mathematical proficiency. With rigor, we will explore and analyze effective strategies to foster each child's growth of number sense and development of mathematical understanding within a numeracy-rich classroom. We will explicitly connect how to integrate literacy skills into mathematics instruction and learning.

Overall Goals

- Teachers will learn to deepen students' understanding of number sense by implementing effective math instructional strategies within a numeracy-rich environment.
- Teachers will strengthen their own knowledge of the progression of numeracy skills within the CC, NBT, and OA domains of the Tennessee Academic Standards.
- Teachers will make important connections to literacy with a particular emphasis on incorporating speaking and listening in mathematics instruction.

Norms for Collaboration

- Keep students at the center
- Be present and engaged
- Monitor air time and share your voice
- Challenge with respect
- Stay solutions oriented
- Risk productive struggle
- Balance urgency and patience

Early Math Messages

Video: Early Math is Big!

While watching the video, think about which messages regarding early mathematics resonate with you. As you view the video, record your important takeaways below. Be prepared to share your takeaways with your table group.

https://www.youtube.com/watch?v=5nmYGWI8UO4

Math in the Early Grades is Developmentally Appropriate.

"Many people believe there is a developmental stage students must go through before they are ready for certain mathematical topics. But these ideas are outdated, as students are as ready as the experiences they have had, and if students are not ready, they can easily become so with the right experiences, high expectations from others, and a growth mindset."

"Students may be unready for some mathematics because they still need to learn some foundational, prerequisite mathematics they have not yet learned, but not because their brain cannot develop the connections because of their age or maturity."

Boaler, Jo. *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages, and Innovative Teaching*. (2016). San Francisco, CA: Jossey-Bass

There is no such thing as a "math brain."

"No one is born lacking the ability to learn math."

"Scientists now know that any brain differences present at birth are eclipsed by the learning experiences we have from birth onward."

Boaler, 2016.

Reflection

How do these quotes compare to your experience? Explain.

Video: Brain Plasticity

As you view the video, record your important takeaways below:
https://www.youtube.com/watch?v=pxru8H6XbR4&list=PLjzmW2yXs7pAPh2ek9CZwP-8w8VaGkC3Y&index=19

Discussion

- Has any information in the opening slides or videos been surprising to you?

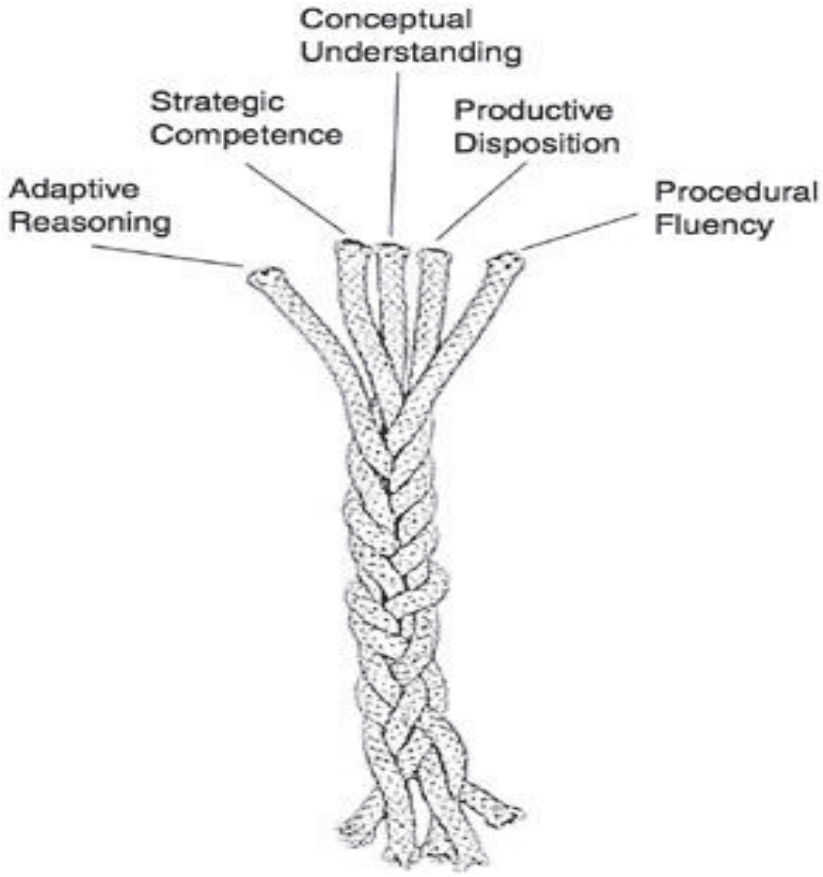
- What are your favorite takeaways?

- What is the early mathematics culture in your school or district?

Note: You can learn more about Jo Boaler's research and resources at youcubed.org.

What is Mathematical Proficiency?

This “rope model” represents the five interwoven strands of mathematical proficiency.



National Research Council. (2001).
Adding it up: Helping children learn mathematics.

- **Conceptual understanding** is understanding mathematical concepts, operations, and relations.
- **Procedural fluency** is the ability to apply procedures accurately, efficiently, flexibly and appropriately.
- **Strategic competence** is the ability to formulate, represent, and solve mathematical problems.
- **Adaptive reasoning** is the capacity for logical thought, reflection, explanation and justification.
- **Productive disposition** is the habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one’s own efficacy.

Tennessee Math Content Standards

In alignment with the five strands of mathematical proficiency, the **Tennessee State Math Standards** focus on a balanced development of conceptual understanding, procedural fluency, and application.

Tennessee Standards for Mathematical Practice (SMPs)

In addition, the Standards for Mathematical Practice are interwoven into the language of our content standards to reflect the proficiency strands of adaptive reasoning, strategic competence, and productive dispositions.

Literacy Skills for Mathematical Proficiency

It is crucial that we recognize and develop the literacy skills that are necessary for achieving mathematical proficiency (i.e., reading, vocabulary, speaking, listening, writing).

Literacy skills for mathematical proficiency include

- using multiple reading strategies,
- understanding and using correct mathematical vocabulary,
- discussing and articulating mathematical ideas, and
- communicating mathematical arguments.

Retrieved from: https://www.tn.gov/assets/entities/sbe/attachments/1-29-16_II_A_Math_Standards_Attachment.pdf

"Brief Version" of the SMPs

Read the "brief version" of the Standards for Mathematical Practice listed below. While reading each description, consider the student actions needed within your grade level in order to demonstrate proficiency with each practice.

SMP #1 - Make sense of problems and persevere in solving them
Students will restate the problem in their own words and then develop a plan to solve the problem. They consistently evaluate their progress and modify their plan as needed. They ask themselves, "Does this make sense?"

SMP #2 - Reason abstractly and quantitatively
Students can take quantitative representation (like a number or equation) and represent it multiple ways (like an image, drawing, or objects). Additionally, they can represent the problem using real-life contexts. Students can also work in the reverse, taking real-life application and putting the information into a visual or quantitative representation.

SMP #3 - Construct viable arguments and critique the reasoning of others
Students can explain how they solved a problem using concise language and applied strategies. They can defend their reasoning as well as recognize their own mistakes. They can question others about their strategies also.

SMP #4 - Model with mathematics
Students can recognize math in real-life and also use math models to solve problems. They may use symbols, pictures, or concrete models to show their thinking and can show relationships to other ideas.

SMP #5 - Use appropriate tools strategically
Students know when to use certain tools to explore and solve problems. They can determine which tool is appropriate and use it appropriately. This could include hands-on tools as well as graphs, charts, visualization, and estimation.

SMP #6 - Attend to precision
Students are efficient and accurate in both their calculations and explanations. They use the correct terminology symbols, and units in their work.

SMP #7 - Look for and make use of structure
Students see and understand how concepts are organized and can break down structures to make sense of new problems. They also look for patterns in problems.

SMP #8 - Look for and express regularity in repeated reasoning
Students look for repeated reasoning in problems to develop shortcuts to make problem solving more efficient. The developed shortcuts are tested and then new learning is generalized to new, more difficult problems.

http://www.doe.in.gov/sites/default/files/standards/process-standards-mathematics-overview.pdf

Discussion

- How do the SMPs relate to grades 1 and 2? What actions should students and teachers be doing in order for students to develop these habits of proficiency?

- How are the SMPs related to the literacy skills for mathematical proficiency?

Module 1:
Teaching Mathematics for Understanding

[TAB PAGE]

Module 1: Teaching Mathematics for Understanding

Objective

- To be able to define what teaching for understanding means in the context of mathematics instruction

Tennessee Standards

The focus of this two day training will be on the Numbers in Base Ten (NBT) and Operations and Algebraic Thinking (OA) standards in both Grade 1 and 2.

Module 1 introduces key understandings to build both teacher content knowledge and pedagogy.

TEAM Alignment

- Presenting Instructional Content
- Questioning
- Teacher Content Knowledge
- Thinking
- Problem Solving

Module Activities

- Participants will read, reflect, and discuss what it means to teach for mathematical understanding
- Participants will sequence student mathematical work to build their math horizon knowledge
- Participants will consider classroom features and routines that promote mathematical understanding
- Participants will connect oral language development to building mathematical understanding by engaging in a vocabulary activity and viewing a number talk strategy

What is mathematical understanding?

Before we begin, take a moment to reflect on what mathematical understanding means to you.

An understanding can never be "covered" if it is to be understood.

-Wiggins and McTighe, 2005, p.229

Understanding is being able to think and act flexibly with a topic or concept. It goes beyond knowing; it is more than a collection of information, facts and data. It is more than being able to follow steps in a procedure. One hallmark of mathematical understanding is a student's ability to justify why a given mathematical claim or answer is true or why a mathematical rule makes sense.

-Council of Chief State School Officers, 2010

Discussion

- How do these two quotes fit with your philosophy of teaching?

- As "understanding" is the foundation of the next two days, what are some specific goals you hope to achieve during this training?

Building Mathematical Knowledge

Read the following quote and highlight the words/phrases that stand out to you.

"Children engaged in the process of problem solving build mathematical knowledge and understanding by grappling with and solving genuine problems, as opposed to completing routine exercises. They use reasoning and proof to make sense of mathematical tasks and concepts and to develop, justify, and evaluate mathematical arguments and solutions. Children create and use representations to reason through problems. They also engage in communication as they explain their ideas and reasoning verbally, in writing, and through representations. Children develop and use connections between mathematical ideas as they learn new mathematical concepts and procedures. They also build connections between mathematics and other disciplines by applying mathematics to real-world situations. By engaging in these processes, children learn mathematics by doing mathematics."

Retrieved from Teaching Student Centered Mathematics
-Van De Walle, Lovin, Karp, & Bay Williams, 2014

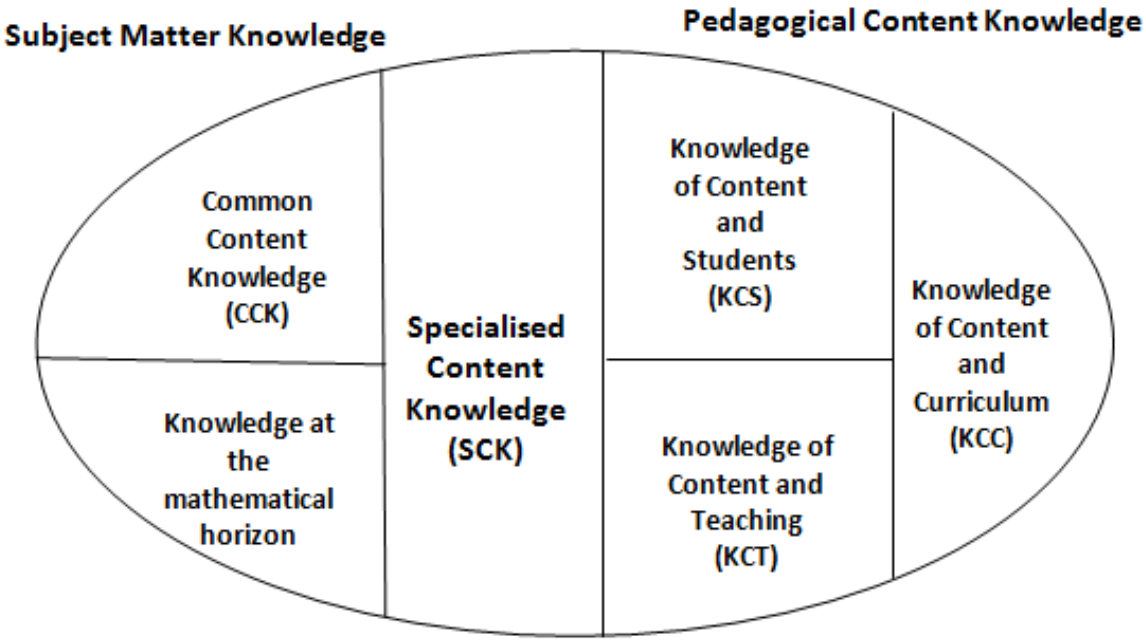
Discussion

- Look at the words or phrases you highlighted. Discuss with your table what you highlighted and why.

- How do these phrases connect to our notion of "understanding"?

MKT: Mathematical Knowledge for Teaching

Mathematical Knowledge for Teaching (MKT) is a kind of complex mathematical understanding, skill, and fluency used in the work of helping others build mathematical knowledge. Beyond being well versed in the content of the curriculum, teachers need significant mathematical skill, perspective, and judgment. Mathematical Knowledge for Teaching is divided into two domains: subject matter knowledge and pedagogical content knowledge. We want to develop each area to improve our teaching effectiveness and ensure students are learning mathematics for understanding. A key component for many early elementary teachers is the Knowledge at the Mathematical Horizon.



Knowledge at the mathematical horizon is an awareness of how mathematical topics are related over the span of mathematics curriculum. First grade teachers, for example, may need to know how the mathematics they teach is related to the mathematics students will learn in third grade to be able to set the mathematical foundation for what will come later. It also includes the vision useful in seeing connections to much later mathematical ideas. Having this sort of knowledge of the mathematical horizon can help in making decisions about how, for example, to talk about the number line.

- Thames & Ball, 2010

Math Horizon Participant Activity

- Use the cards provided to think about the mathematical horizon on which our students develop number sense.
- Based on what you know about the expectations for learning from kindergarten to third grade standards, order the cards from introductory understanding of the concept to the most developed understanding and be prepared to justify your decisions.
- When everyone at your table has come to an agreement on the order of the cards, tape your cards to the chart paper.

Gallery Walk

- Circulate and analyze the sequence of tasks/activities
- Visit two or three groups' sequence of activities. Consider the order of the tasks/activities as identified by other groups.
- What is the same and what is different among the different sequences on display?
- Leave feedback for each group by recording a "wondering" and "noticing" on sticky notes.

Discussion

- Why is the knowledge of the mathematical horizon important as an educator?

- How could this knowledge guide you in the decisions you make for your classroom?

- How could deepening your knowledge of the mathematical horizon help you teach mathematics for understanding?

Common addition and subtraction situations.

Note: Beginning in kindergarten, students should be exposed to all addition and subtraction problem types. The different shading of the boxes in the table below indicates when mastery of that problem type is expected.

- The darkest shading indicates the problem types to be mastered at the end of kindergarten.
- The lighter shading indicates the problem types to be mastered at the end of first grade.
- The unshaded boxes indicate the problem types to be mastered at the end of second grade.

	Results Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take From	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two of them. Then there were three apples. How many apples were on the table before? ? – 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**
Put together/take apart	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many can she put in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 4 + 1, 5 = 1 + 4 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	("How many more?" Version) Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? 2 + ? = 5, 5 – 2 = ?	(Version with "More") Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "More") Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5
	("How many fewer?" Version) Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "Fewer") Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "Fewer") Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

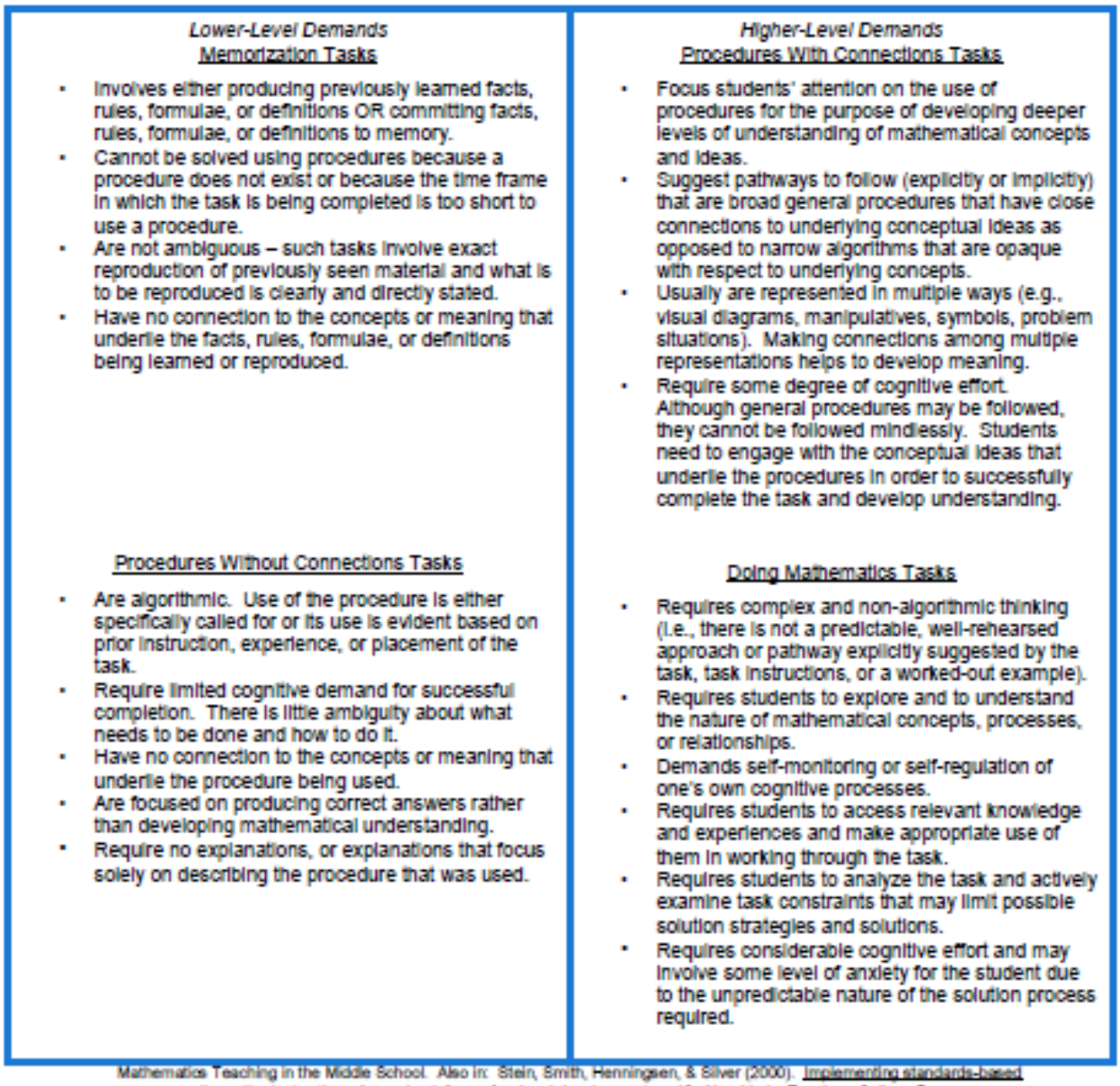
Classroom Planning Tool

The template below can be used as a planning tool to help balance the types of situational problems that students have the opportunity to solve during units of instruction.

Action		Result Unknown	Change Unknown	Start Unknown
	Join			
	Separate			
Nonaction		Whole Unknown	Part Unknown	
	Part-Part-Whole			
			Difference Unknown	Greater Unknown	Lesser Unknown
	Compare	"How many more?"			
		"How many fewer?"			

Classroom Planning Tool
Task Analysis Guide (TAG)

Another excellent planning tool is the Task Analysis Guide (TAG) that was introduced in previous summer trainings. The TAG can help us make sure that there is an appropriate balance of high-level instructional tasks.



Lower-Level Demands

Memorization Tasks

- Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.

Procedures Without Connections Tasks

- Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers rather than developing mathematical understanding.
- Require no explanations, or explanations that focus solely on describing the procedure that was used.

Higher-Level Demands

Procedures With Connections Tasks

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.

Doing Mathematics Tasks

- Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).
- Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships.
- Demands self-monitoring or self-regulation of one's own cognitive processes.
- Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Mathematics Teaching in the Middle School. Also in: Stein, Smith, Henningsen, & Silver (2000). Implementing standards-based mathematics instruction: A casebook for professional development, p. 16. New York: Teachers College Press.

Classrooms that Promote Mathematical Understanding

What are characteristics of a classroom that promotes mathematical understanding?

"The essence of developing rational understanding is to keep the children's ideas at the forefront of classroom activities by emphasizing the process standards, mathematical proficiencies and the Standards for Mathematical Practice. This requires that the teacher create a classroom culture in which children can learn from one another."

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

"The purpose of discourse is not for children to state their answers and get validation from the teacher, but to engage all learners and keep the cognitive demand high."

-Breyfogle & Williams, 2009

Participant Activity: Take a moment and read the two quotes above. Discuss these quotes with your table. As a group decide on a list of features that are a "must do" in a classroom that promotes mathematical understanding. Chart the group's "must" classroom characteristics and be prepared to share with the whole group.

Below is a list of features of a mathematics classroom that promote understanding:

- *Children's ideas are key*- mathematical ideas expressed by children are important and have the potential to contribute to everyone's learning. Learning mathematics is about coming to understand the ideas of the mathematical community.

- *Opportunities for children to talk about mathematics are common*- Learning is enhanced when children are engaged with others who are working on the same ideas. Encouraging student to student dialogue can help children think of themselves as capable of making sense of mathematics. Children are also more likely to question each other's ideas than the teacher's ideas.

- *Multiple approaches are encouraged*- Children must recognize that there is often a variety of methods that will lead to a solution. Respect for the ideas shared by others is critical if real discussion takes place.

- *Mistakes are good opportunities for learning*- Children must come to realize that errors provide opportunities for growth as they are uncovered and explained. Trust must be established with an understanding that it is okay to make mistakes. Without this trust, many ideas will never be shared.

- *Math makes sense*- Children must come to understand that mathematics makes sense. Teachers should resist always evaluating children's answers. In fact, when teachers routinely respond with "Yes, that's correct," or "No, that's wrong," children will stop trying to make sense of ideas in the classroom and discussion and learning will be curtailed.

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

Reflection

- What are the differences and similarities between the above list and the list your group created?

- As you think about your own classroom, is there anything that you would like to refine in your own teaching practices?

"Mistakes are not only opportunities for learning, as students consider the mistakes but also time when our brain grows."

- Boaler, 2016.

NOTE: It is important for students to see counterexamples, so purposefully point them out and have discussions about them. These discussions allow children to explore misunderstandings.

Video: My Favorite No

https://www.teachingchannel.org/videos/class-warm-up-routine

- As you watch the video, what are some strategies this teacher uses that makes this strategy effective?

- What connections can you make to developing oral language skills and the Strands of Mathematical Proficiency?

Small Group Activity

With your small group, discuss some of the favorite no's you may address with your students on these various problems.

- Write Two Hundred Eighty-Three in standard form
- 60+8 = _____ tens and _____ ones
- 5 tens and 3 ones = 13 ones and ____ tens
- 17 + 18
- 6 -☐= 3
- 58 + 29
- Write Three Hundred Four in standard form

Oral Language Needs for Developing Mathematical Understanding

"Oral language development is one of the most important, yet basic, foundational skills of children. If children come to school with well-developed oral language, it must be expanded. If children come to school with underdeveloped oral language, it must be developed."
"It is time we reflect on and explore how oral language can affect not only reading proficiency but math proficiency as well."

Pace, M. H., & Ortiz, E. (2015). Oral Language Needs: Making Math Meaningful. *Teaching Children Mathematics*, 494-500.

Activity: Vocabulary Strategy

In the article quoted above, the authors describe a mathematics vocabulary strategy that helps young students make meaning of mathematical vocabulary through pictures, words, and oral communication.

(Pace & Ortiz, 2015)

Read the excerpt below that describes how the author implemented the vocabulary strategy in her classroom:

"I created an introductory mathematics lesson focusing on the word joining. After many modeling examples, students were ready to create the vocabulary chart by creating their own pictures to explain what the word joining meant. They were asked to take an index card back to their seat and draw a picture that would explain to the class what joining meant. I explained to them that they must be able to tell the class about their picture and why it represented *joining*."

"After ten minutes of watching my little mathematicians draw furiously, we started our group sharing. One by one, hands flew into the air, eager to share their work. Students were able to define the word *join* by creating a meaningful picture. At this point in the lesson, I could tell no observable difference between the varying academic levels of my students."

"In my classroom, students were making sense of the term *joining* in their own way, making their understanding meaningful and observable trhough their pictures and dialogue."

(Pace & Ortiz, 2015)

Discussion

- How does this vocabulary strategy strengthen student mathematical understanding?

- What activities or strategies do you do in your classroom to help your students make sense of mathematical vocabulary and develop oral language?

Vocabulary Chart Activity

- Split into grade level groups
- Count off by 4's. Gather each number together.
- Each group will utilize the strategy from the article and plan out various ways in which they would present it to their students. Be specific: what types of experiences do you want them to have.
- Draw your own visual for the word you are given and display on chart paper.
- Suggested vocabulary
    - 1st grade- Greater than, less than, Expression, Equal
    - 2nd grade- Equation, Compose, Decompose

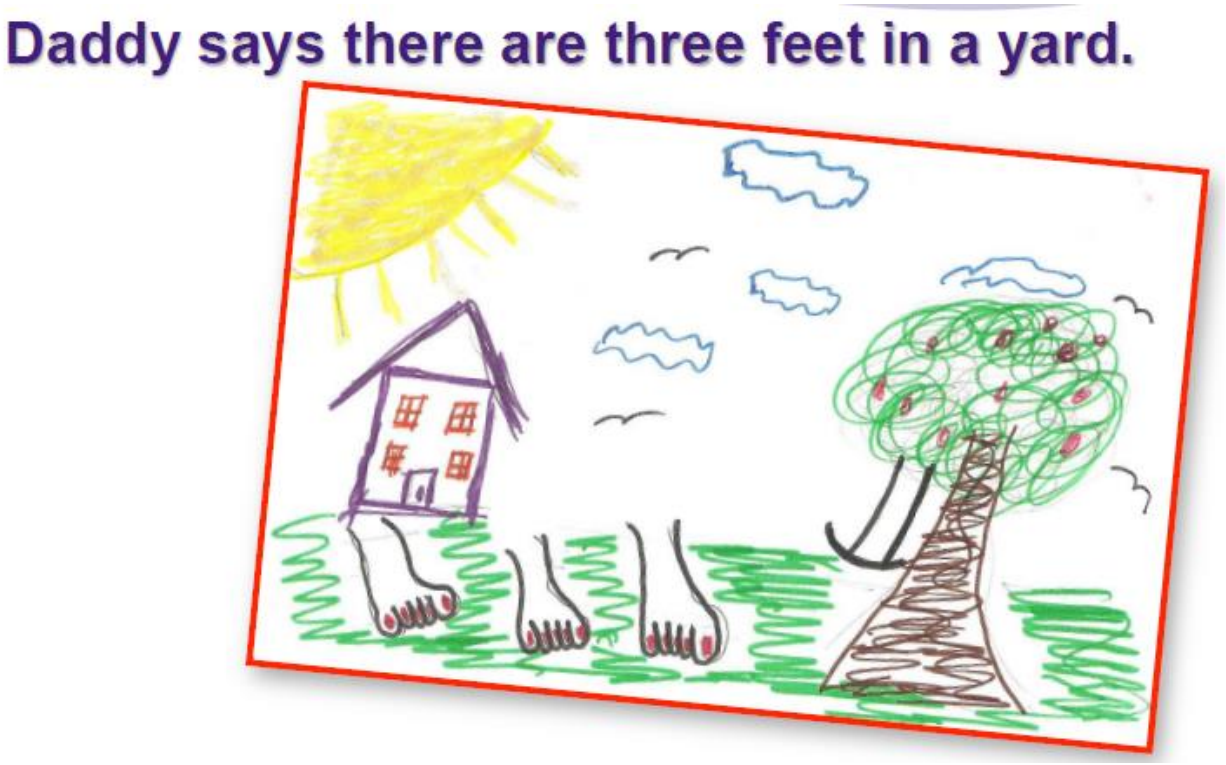
Gallery Walk

- What do you notice about other participants' depictions of the vocabulary words?

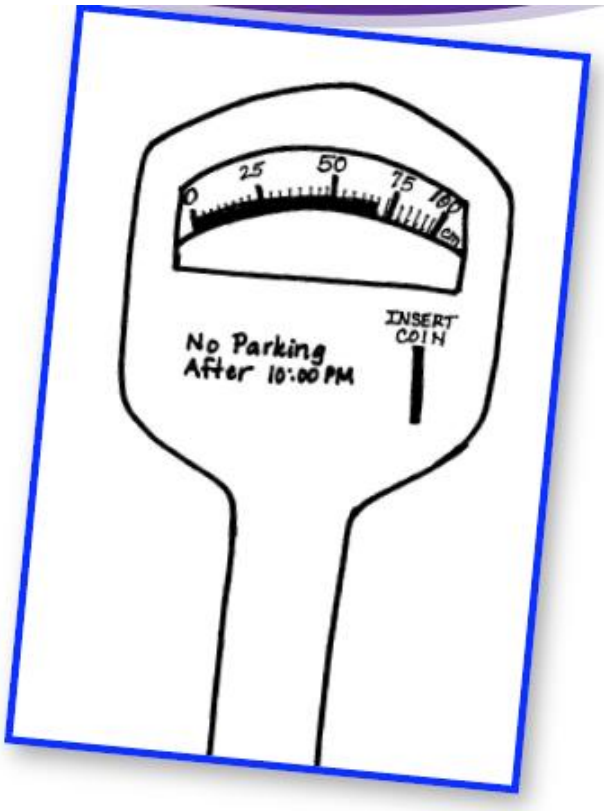
- How is this type of activity beneficial for a classroom promoting mathematical understanding?

Vocabulary in Mathematics

Without specific mathematics vocabulary instruction, students may become confused as some words are shared by a content area and everyday English, but have different meanings.



The teacher said there are 100 centimeters in a meter.



My teacher says the Earth is like a giant sphere.

My teacher said we needed to find the volume of the cereal box.

Mom says a rectangular prism has 6 faces.

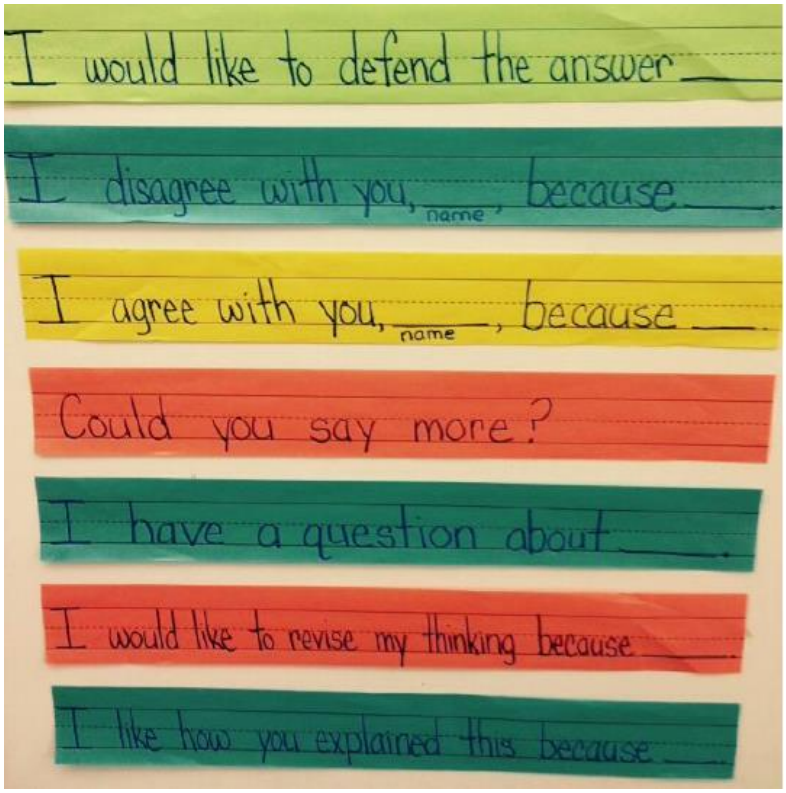
Retrieved from https://www.mentoringminds.com/documents/VocabularyinMathematics.pdf

Supporting Oral Language Development with Number Talks

Video: Language Supports for Number Talks

https://www.teachingchannel.org/videos/sentence-frames-ousd

As you view the video, record your key takeaways below:



Discussion:

- How can sentence frames empower students during number talks?

- How are the sentence frames related to Accountable Talk?

- Create some sentence frames with your table group.

Module 1 Closing Reflection

- How has this module changed the way you think about mathematics instruction?

- What will you do differently as a result?

- What are you still wondering about?

Module 1 Collaborative Notes

Ideas for collaboration: vocabulary strategies or routines; math talk or accountable talk strategies; resources on standards for mathematical practice; resources for addition and subtraction problem types.

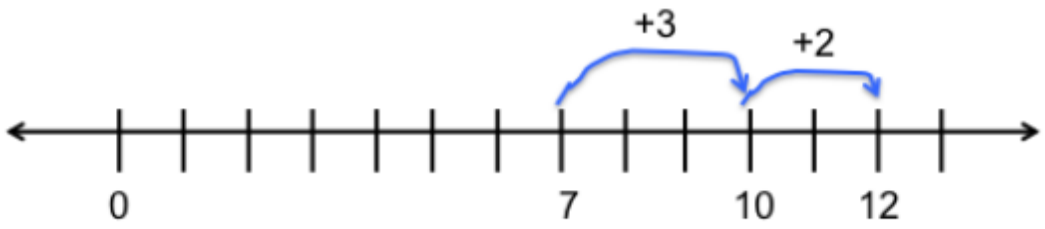
Module 1 Appendix

TAB

PAGE

Jake has some animal books. He has 7 car books. Altogether he has 12 books. How many of the books are animal books?

Student Work (number line was already drawn):



___ + 7 = 12

You can turn this around and write:

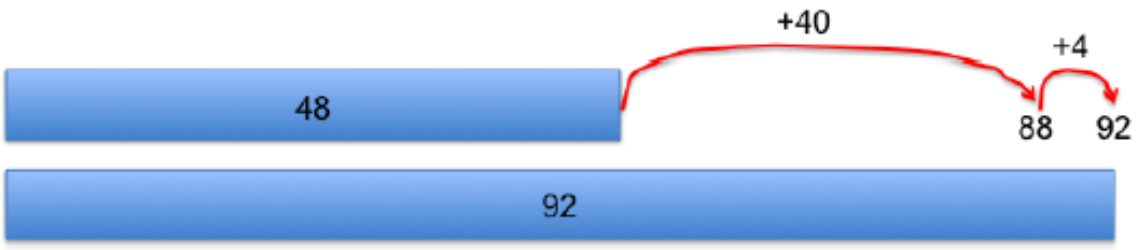
7 + ___ = 12

Start at 7 and count up 5 more:

7 + 5 = 12

Mary has 48 rings and John has 92 rings. How many more rings does John have than Mary?

Student Work (this problem has a second part below):



92 = 48 + <u>44</u>

John has 44 more rings than Mary.

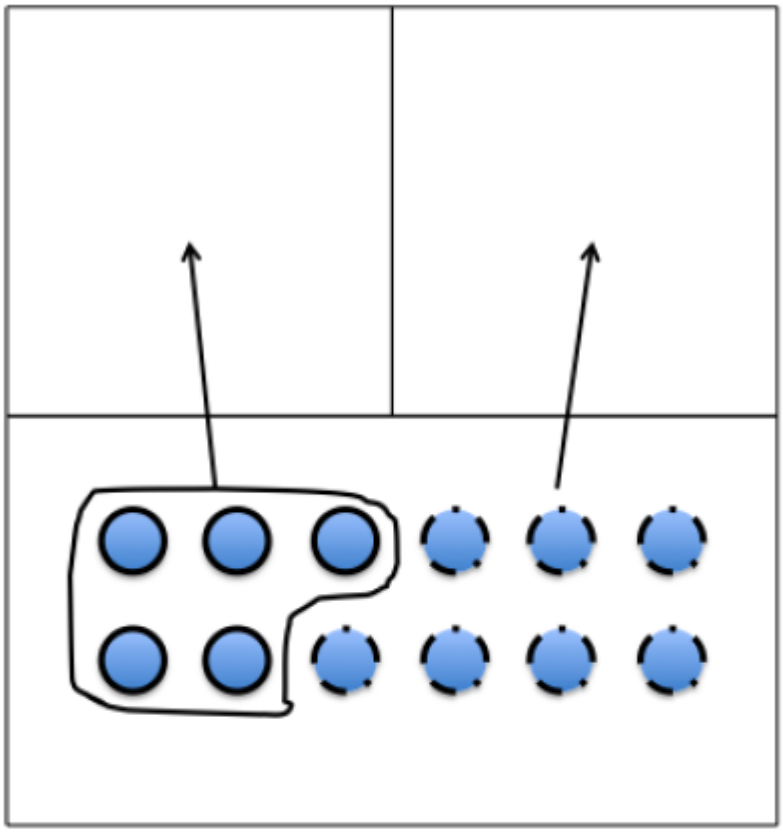
How many more does Mary need to equal John's amount of rings?

48 + <u>40</u> + <u>4</u> = 92

48 + <u>44</u> = 92

Sam has 12 pieces of candy. He eats 5 pieces. How many pieces of candy does he have left?

Student Work:

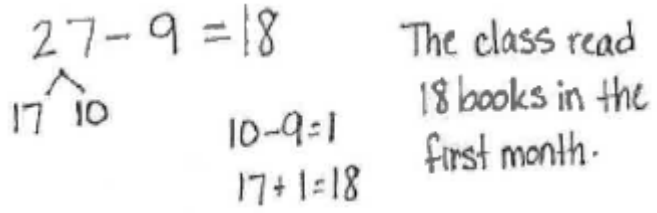


12 – 5 = ___

12 – 5 = 7

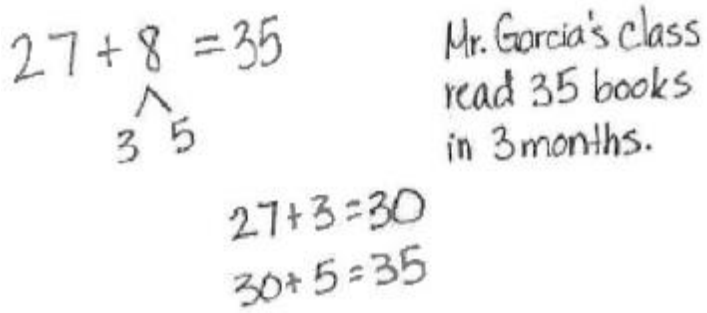
4. Mr. Garcia checked out 27 library books for his class. The class read some the first month and the remaining 9 books the second month.
   a. Use words, pictures, or numbers to find out how many books the class read in the first month.

Student work:



b. During the third month, Mr. Garcia checked out 8 more books and his class read them all. Use words, pictures, or numbers to show how many library books Mr. Garcia's class read in all 3 months.

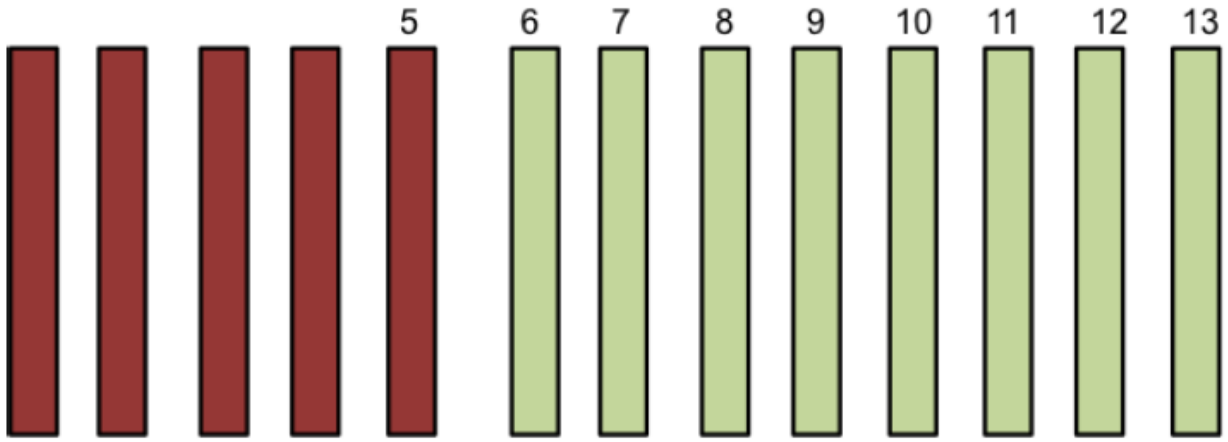
Student Work:



(Retrieved from engageny.org)

Joe has 5 red pencils and 8 green pencils in the case. How many pencils does he have in the case?

Student Work:

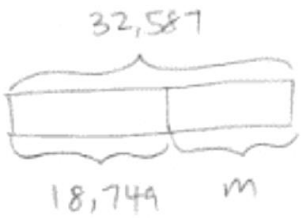


5 + 8 = ____

8 + 5 = 13 or 5 + 8 = 13

Draw a tape diagram to represent each problem. Use numbers to solve, and write your answer as a statement.

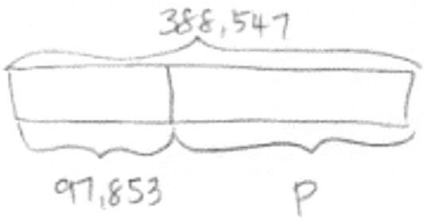
1. Sean's school raised $32,587. Leslie's school raised $18,749. How much more money did Sean's school raise?



32,587
- 18,749
13,838

Sean's school raised $13,838 more money than Leslie's school.

2. At a parade, 97,853 people sat in bleachers, and 388,547 people stood along the street. How many fewer people were in the bleachers than standing on the street?



388,547
- 97,853
290,694

There were 290,694 fewer people in the bleachers.

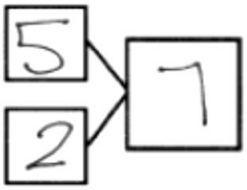
(Retrieved from engageny.org)

1.

5 + 2 = 7

5 balls are here. 2 more roll over. Now, there are 7 balls.

Make a number bond to match the story.

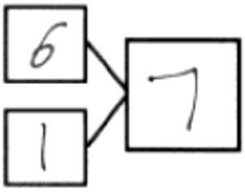


2.

6 1 = 7

6 frogs are here. 1 more hops over. Now, there are 7 frogs.

Make a number bond to match the story.



(Retrieved from engageny.org)

5. Martha answered the problem 400 – 378 incorrectly. She does not understand her mistake.

a. Explain to Martha what she did wrong using place value language.

Explanation:

Martha changed 1 hundred for 10 tens and 10 ones. It should have been changed for 9 tens and 10 ones.

b. Model an alternate strategy for 400 – 378 to help Martha avoid making this mistake again.

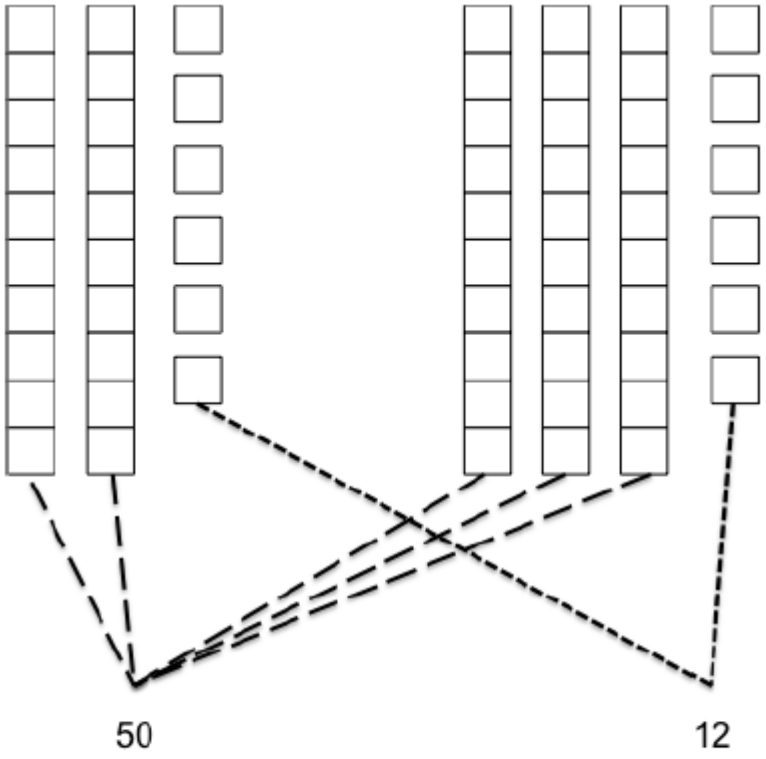
Martha can take 1 from each number, then subtract.

$$\begin{array}{r} 399 \\ -377 \\ \hline 22 \end{array}$$

(Retrieved from engageny.org)

Mary has 26 cookies. She gets 36 more cookies. How many cookies does she have altogether?

Student Work:



26 + 36 = ___

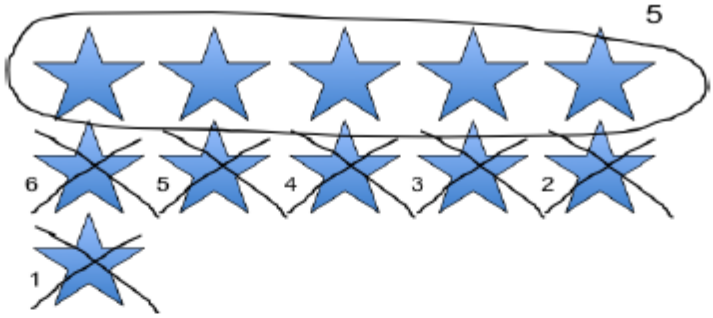
20 + 30 = 50 *and* 6 + 6 = 12

50 + 12 = ___

60 + 2 = 62

Tabitha has 11 star stickers. She puts some of the star stickers on her paper. She has 5 star stickers left. How many did she put on her paper?

Student Work:

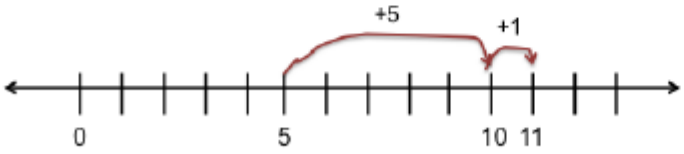


11 – ____ = 5

Turn around and write 11 – 5 = ___

11 – 5 = 6 so 11 – 6 = 5

Can you show a second way of solving this problem?

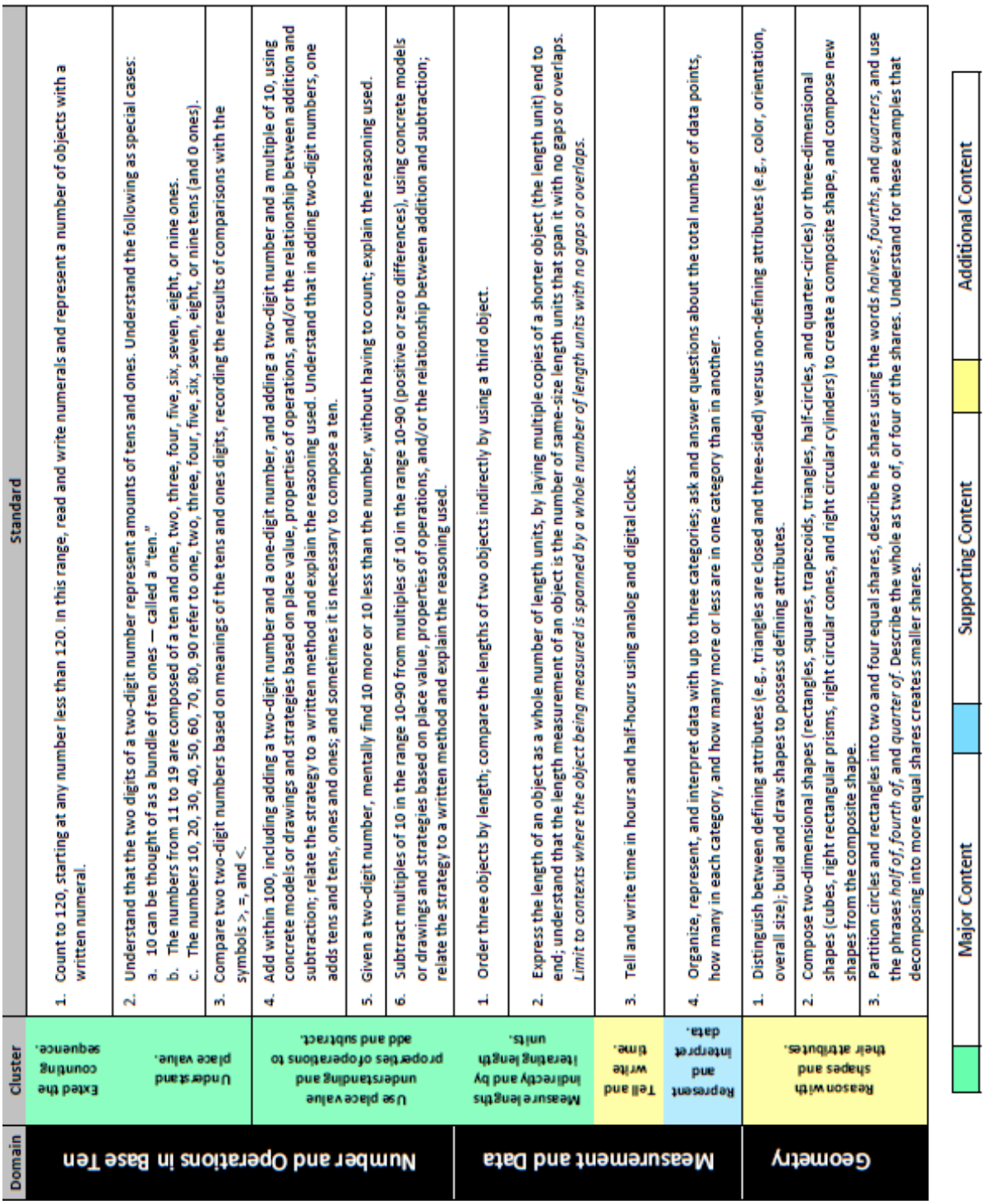


5 + ___ = 11, so 5 + 6 = 11 or 11 – 6 = 5

2016 Tennessee Math Content Standards, Grade 1

Domain	Cluster	Standard
Operations and Algebraic Thinking	Represent and solve problems involving addition and subtraction.	1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
		2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
	Understand and apply properties of operations and the relationship between addition and subtraction.	3. Apply properties of operations as strategies to add and subtract. *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)* (Students need not use formal terms for these properties.)
		4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*
	Add and subtract within 20.	5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
		6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
	Work with addition and	7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*
		8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ▯ – 3, 6 + 6 = ▯.*

2016 Tennessee Math Content Standards, Grade 1



Domain	Cluster	Standard
Number and Operations in Base Ten	Extend the counting sequence.	1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
	Understand place value.	2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases: a. 10 can be thought of as a bundle of ten ones — called a "ten." b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
		3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
	Use place value understanding and properties of operations to add and subtract.	4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
		5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
		6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
Measurement and Data	Measure lengths indirectly and by iterating length units.	1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
		2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*
	Tell and write time.	3. Tell and write time in hours and half-hours using analog and digital clocks.
	Represent and interpret data.	4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Geometry	Reason with shapes and their attributes.	1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
		2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.
		3. Partition circles and rectangles into two and four equal shares, describe he shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

Major Content | Supporting Content | Additional Content

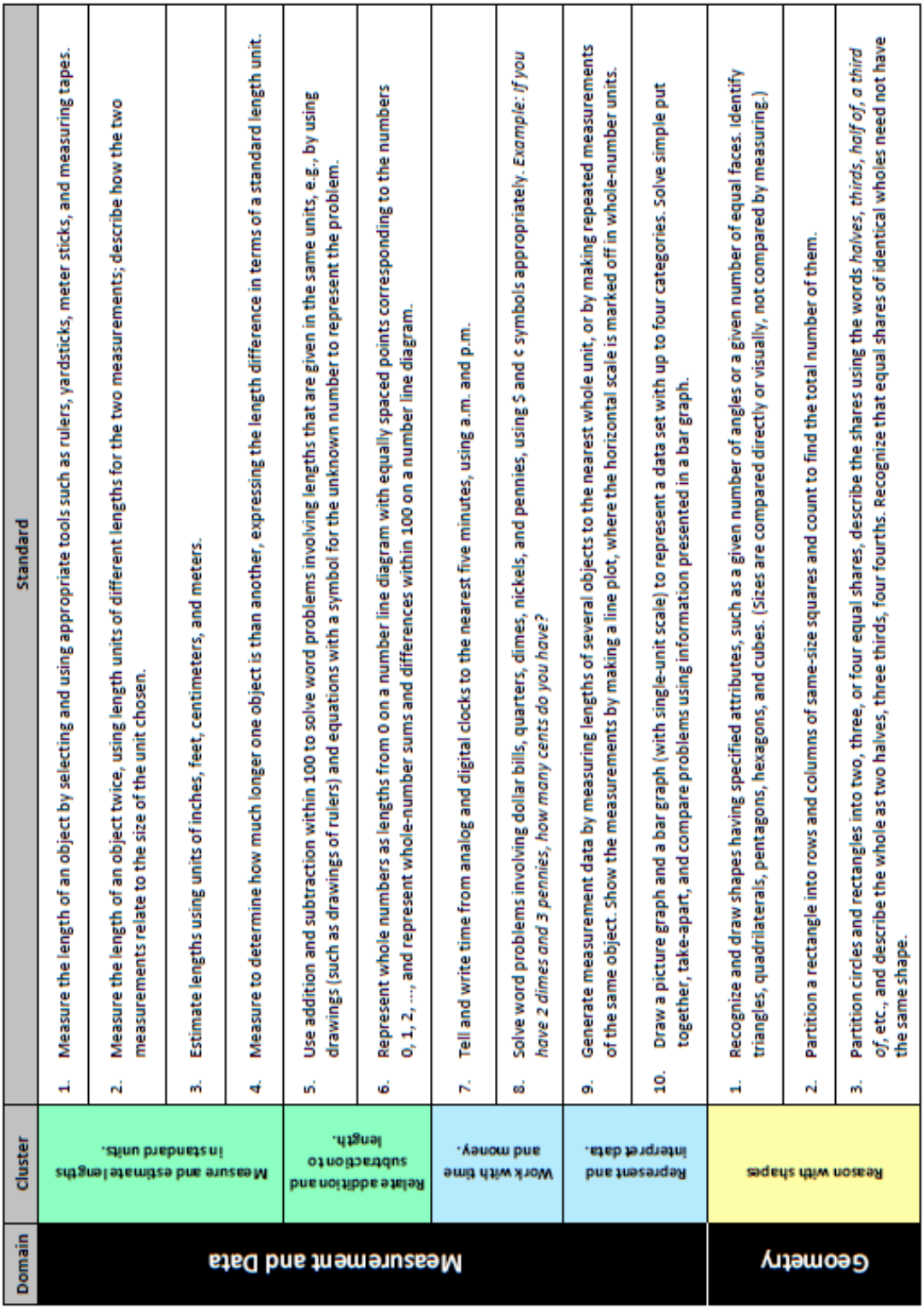
TN Department of Education

2016 Tennessee Math Content Standards, Grade 2

Domain	Cluster	Standard
Operations and Algebraic Thinking	Represent and solve problems involving addition and subtraction.	1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
	Add and subtract within 20.	2. Fluently add and subtract within 20 using mental strategies.2 By end of Grade 2, know from memory all sums of two one-digit numbers.
	Work with equal groups of objects to gain foundations for multiplication.	3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
		4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Number and Operations in Base Ten	Understand place value.	1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens — called a "hundred." b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
		2. Count within 1000; skip-count by 5s, 10s, and 100s.
		3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
		4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
	Use place value understanding and properties of operations to add and subtract.	5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
		6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
		7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
		8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
		9. Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)

TN Department of Education

2016 Tennessee Math Content Standards, Grade 2



2016 Tennessee Math Content Standards, Grade 3

Domain	Cluster	Standard
Operations and Algebraic Thinking	Represent and solve problems involving multiplication and division.	1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
		2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
		3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1
		4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = ▯ ÷ 3, 6 × 6 = ?.*
	Understand properties of multiplication and the relationship between multiplication and division.	5. Apply properties of operations as strategies to multiply and divide.2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3× 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)* (Students need not use formal terms for these properties.)
		6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*
	Multiply and divide within 100.	7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
	Solve problems involving the four operations, and identify and explain patterns in arithmetic.	8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).)
		9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

2016 Tennessee Math Content Standards, Grade 3

Domain	Cluster	Standard
Number and Operations in Base Ten	Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)	1. Use place value understanding to round whole numbers to the nearest 10 or 100.
		2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
		3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.
Number and Operations—Fractions*	Develop understanding of fractions as numbers.	1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
		2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line. b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
		3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 =2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. *Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

2016 Tennessee Math Content Standards, Grade 3

Domain	Cluster	Standard
Measurement and Data	Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.	1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
		2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).6 Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
	Represent and interpret data.	3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
		4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units— whole numbers, halves, or quarters.
	Geometric measurement: understand concepts of area and relate area to multiplication and to addition.	5. Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
		6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
		7. Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
	Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.	8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
Geometry	Reason with Shapes and their attributes.	1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
		2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Major Content	Supporting Content	Additional Content

2016 Speaking and Listening Standards, Grade 1

	Standard
Comprehension and Collaboration	1. Participate in collaborative conversations with diverse partners about *grade 1 topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., listening to others with care, speaking one at a time about the topics and texts under discussion). b. Build on others' talk in conversations by responding to the comments of others through multiple exchanges. c. Ask questions to clear up any confusion about the topics and texts under discussion. 2. Ask and answer questions about key details in a text read aloud or information presented orally or through other media. 3. Ask and answer questions about what a speaker says in order to gather additional information or clarify something that is not understood.
Presentation of Knowledge and Ideas	4. Describe people, places, things, and events with relevant details, expressing ideas and feelings clearly. 5. Add drawings or other visual displays to descriptions when appropriate to clarify ideas, thoughts, and feelings. 6. Produce complete sentences when appropriate to task and situation. (See grade 1 Language standards 1 and 3 on page 16 for specific expectations.)

TN Department of Education

2016 Speaking and Listening Standards, Grade 2

Standard	
Comprehension and Collaboration	1. Participate in collaborative conversations with diverse partners about *grade 2 topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., gaining the floor in respectful ways, listening to others with care, speaking one at a time about the topics and texts under discussion). b. Build on others' talk in conversations by linking their comments to the remarks of others. c. Ask for clarification and further explanation as needed about the topics and texts under discussion. 2. Recount or describe key ideas or details from a text read aloud or information presented orally or through other media. 3. Ask and answer questions about what a speaker says in order to clarify comprehension, gather additional information or deepen understanding of a topic or issue.
Presentation of Knowledge and Ideas	4. Tell a story or recount an experience with appropriate facts and relevant, descriptive details, speaking audibly in coherent sentences. 5. Create audio recordings of stories or poems; add drawings or other visual displays to stories or recounts of experiences when appropriate to clarify ideas, thoughts, and feelings. 6. Produce complete sentences when appropriate to task and situation in order to provide requested detail or clarification. (See grade 2 Language standards 1 and 3 on page 26 for specific expectations.)

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Accountable Talk® Features and Indicators

Accountability to the Learning Community

- Active participation in classroom talk
- Listen attentively
- Elaborate and build on each other's ideas
- Work to clarify or expand a proposition

Accountability to Knowledge

- Specific and accurate knowledge
- Appropriate evidence for claims and arguments
- Commitment to getting it right

Accountability to Rigorous Thinking

- Synthesize several sources of information
- Construct explanations and test understanding of concepts
- Formulate conjectures and hypotheses
- Employ generally accepted standards of reasoning
- Challenge the quality of evidence and reasoning

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TNCore

Accountable Talk® Moves

Talk Move	Function	Example
To Ensure Purposeful, Coherent, and Productive Group Discussion		
Marking	Direct attention to the value and importance of a student's contribution.	That's an important point.
Challenging	Redirect a question back to the students or use students' contributions as a source for further challenge or query.	Let me challenge you: Is that always true?
Revoicing	Align a student's explanation with content or connect two or more contributions with the goal of advancing the discussion of the content.	S: 4 + 4 + 4. You said three groups of four.
Recapping	Make public in a concise, coherent form, the group's achievement at creating a shared understanding of the phenomenon under discussion.	Let me put these ideas all together. What have we discovered?

TN Department of Education

Accountable Talk® Moves

Talk Move	Function	Example
	To Ensure Purposeful, Coherent, and Productive Group Discussion	
Marking	Direct attention to the value and importance of a student's contribution.	It is important to say describe to compare the size of the pieces and then to look at how many pieces of that size.
Challenging	Redirect a question back to the students or use students' contributions as a source for further challenge or query.	Let me challenge you: Is that always true?
Revoicing	Align a student's explanation with content or connect two or more contributions with the goal of advancing the discussion of the content.	You said 3, yes there are three columns and each column is 1/3 of the whole
Recapping	Make public in a concise, coherent form, the group's achievement at creating a shared understanding of the phenomenon under discussion.	Let me put these ideas all together. What have we discovered?
	To Support Accountability to Community	
Keeping the Channels Open	Ensure that students can hear each other, and remind them that they must hear what others have said.	Say that again and louder. Can someone repeat what was just said?
Keeping Everyone Together	Ensure that everyone not only heard, but also understood, what a speaker said.	Can someone add on to what was said? Did everyone hear that?
Linking Contributions	Make explicit the relationship between a new contribution and what has gone before.	Does anyone have a similar idea? Do you agree or disagree with what was said? Your idea sounds similar to his idea.
Verifying and Clarifying	Revoice a student's contribution, thereby helping both speakers and listeners to engage more profitably in the conversation.	So are you saying..? Can you say more? Who understood what was said?
	To Support Accountability to Knowledge	
Pressing for Accuracy	Hold students accountable for the accuracy, credibility, and clarity of their contributions.	Why does that happen? Someone give me the term for that.
Building on Prior Knowledge	Tie a current contribution back to knowledge accumulated by the class at a previous time.	What have we learned in the past that links with this?
	To Support Accountability to Rigorous Thinking	
Pressing for Reasoning	Elicit evidence to establish what contribution a student's utterance is intended to make within the group's larger enterprise.	Say why this works. What does this mean? Who can make a claim and then tell us what their claim means?
Expanding Reasoning	Open up extra time and space in the conversation for student reasoning.	Does the idea work if I change the context? Use bigger numbers?

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Module 2:

Developing Early Number Concepts and Number Sense

Module 2: Developing Early Number Concepts and Number Sense

Objective

- To introduce a scope and sequence of early number concepts and number sense. This module will look at the development of number ideas for numbers up to 20.

Tennessee Standards

1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*
1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
2.OA.4 Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

TEAM Alignment

- Standards and Objectives
- Presenting Instructional Content
- Activities and Materials
- Questioning
- Teacher Content Knowledge
- Thinking
- Problem Solving

Participant Activities:

- Participants will read, reflect, and discuss ways to build number sense.
- Participants will learn classroom activities for subitizing (spatial relationships) and building understanding of number relationships within sums of 20.
- Participants will learn activities to help students develop conceptual and procedural understanding of part-part-whole relationships and patterns within 100.

Early Counting

"No matter what prior experiences children have had before coming to school, we need to strive to help all children develop the following four interrelated aspects of early numerical knowledge."

Clements & Sarama, 2009

4 Aspects of Early Numerical Knowledge

- Number Sequence: the names and the ordered list of number words.
- One to one correspondence: counting objects by saying number words in a one to one correspondence with the objects.
- Cardinality: understanding that the last number word said when counting tells how many objects have been counted.
- Subitizing: quickly recognizing and naming how many objects are in a small group without counting. Young children can recognize and name quantities of objects that are less than four without counting.

Clements & Sarama, 2007

Assessment Note

Children have opportunities to develop the four aspects of early numerical knowledge within a smaller set of numbers in pre-K and kindergarten. For children who struggle with grade 1 and grade 2 math content, it is important to assess their knowledge of these four aspects as they may have unfinished learning in those foundational aspects from pre-K and kindergarten. This is when the RTI^2 model will help students fill in the gaps in their foundations through intervention at the Tier II or Tier III level.

Developing Number Sense by Building Number Relationships

Video: Number Sense

https://www.youtube.com/watch?v=Jeel4Qjow4s

As you view the Number Sense video, record your key takeaways below:

Number Sense is not a set of skills that children can develop in a short period of time. It is something that grows and develops over time. These foundational ideas can all be extended to larger numbers, operations, basic facts, and computation.

> "Number sense is a good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms."

Howden, 1989

Turn and Talk

- What are your thoughts on the video and two quotes above?

Relationships between Numbers 1 through 10

Once children have acquired the first three aspect of Early Numerical Knowledge (Number Sequence, One to One correspondence, and Cardinality), too often teachers move directly from the beginning ideas of counting to addition and subtraction, leaving children with a very limited collection of ideas about numbers to bring to these new topics. The result is that often children continue to count by ones to solve simple story problems and have difficulty mastering basic facts. Emphasizing number relationships is key to helping children fully develop number sense.

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

Small Group Discussion

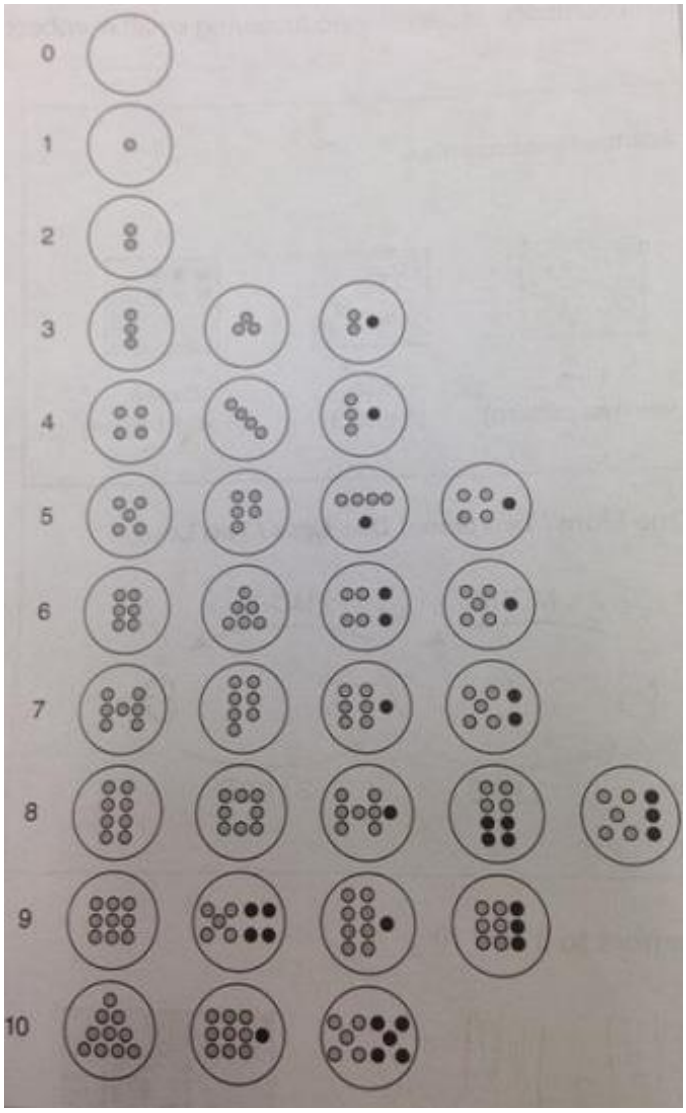
- What does the above paragraph say is the result of children moving too quickly from the beginning ideas of counting straight to addition and subtraction?

- Reflecting on your own practices in the classroom, what is your response to the above paragraph?

Spatial relationships

Before moving from the concepts of counting into addition and subtraction concepts, students need opportunities with the fourth aspect of early numerical knowledge, subitizing. Children can learn to recognize sets of objects in patterned arrangements and tell how many without counting. The figure below shows a set of "dot plates" that can be used to promote subitizing.

DOT PLATES



- Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Participant Activity: Make a set of dot plates

Using the materials provided, construct your own set of Dot Plates by choosing five images from the dot plate graphic. These will be used in the activities throughout this professional development and in your classroom. Pay close attention to the coloring of various patterns.

Classroom Activities to Promote Subitizing (Spatial Relationships)

Student Activity: LEARNING PATTERNS

Materials: 10 counters and piece of cardstock per child

To introduce the patterns, provide each child with about 10 counters and a piece of cardstock or paper as a mat.

1. Hold up a dot plate for about 5 seconds and say, "Make the pattern you saw on the plate using the counters on the mat. How many dots did you see? What did the pattern look like?
2. Spend some time discussing the configuration of the pattern and the number of dots.
3. Show the plate so they can self-check.
4. Continue with a few patterns each day.

Van de Walle, Lovin, Karp, & Bay-Williams. *Teaching Student Centered Mathematics*, 2014

Student Activity: DOT PLATE FLASH

1. Hold up a dot plate for only 1-3 seconds and say, "How many dots do you see? What did the pattern look like?"
2. Include easy patterns first and then add more dots as children's confidence builds.
3. Initially, you may need to show a plate a second time so that children can get a second look.
4. Children like to see how quickly they can recognize and say how many dots.
5. Children can also flash dot plates to each other as a workstation activity.

Van de Walle, Lovin, Karp, & Bay-Williams. *Teaching Student Centered Mathematics*, 2014

Video: Subitizing Activity

https://www.youtube.com/watch?v=3UYn0w-uhZg

Video Notes:

Reflection

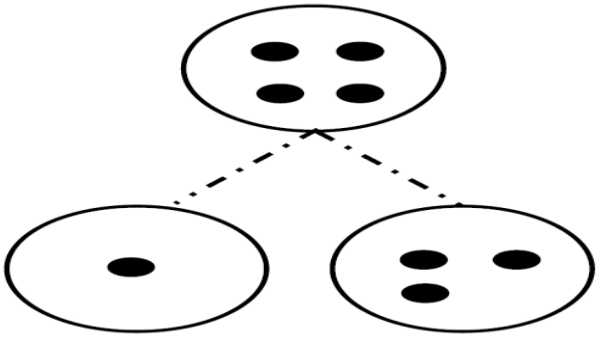
What do you notice about this video and how the images are displayed?

How do you these activities help promote mathematical understanding?

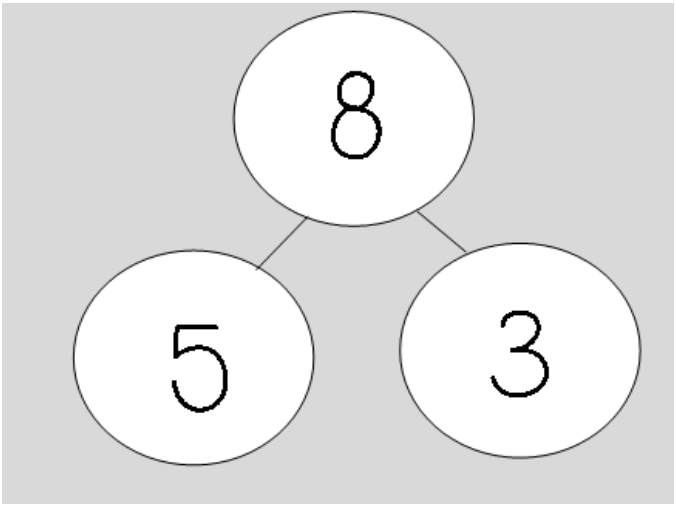
How subitizing activities develop number sense

Journey of the Number Bond

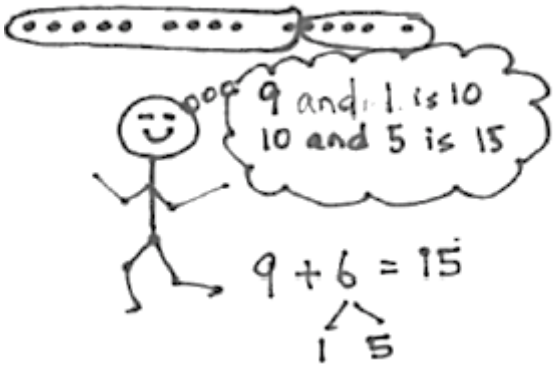
As students become more familiar with dot patterns, teachers may begin to make connections to Number Bonds. A students' ability to understand and manipulate number bonds supports their long-term development of number sense.



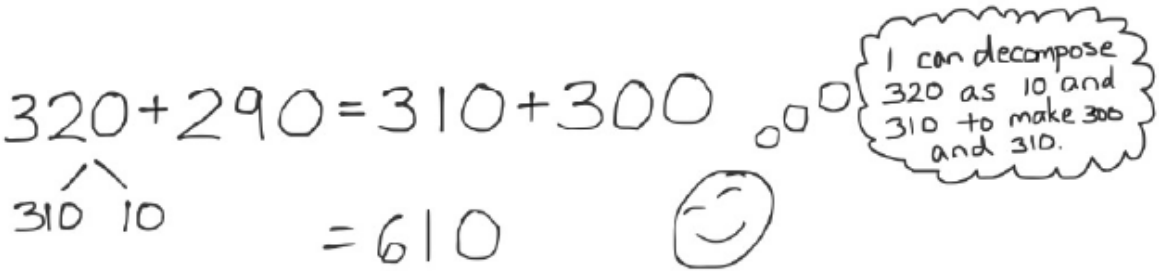
Number Bond with Dot Pattern



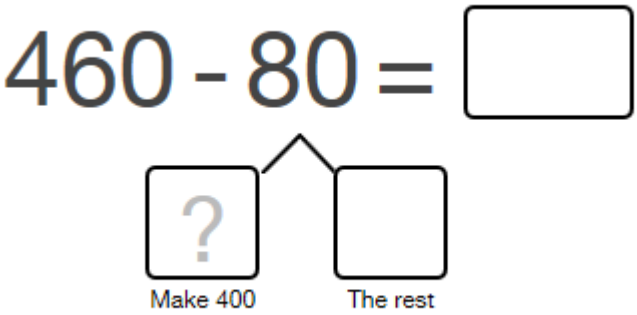
Traditional Number Bond



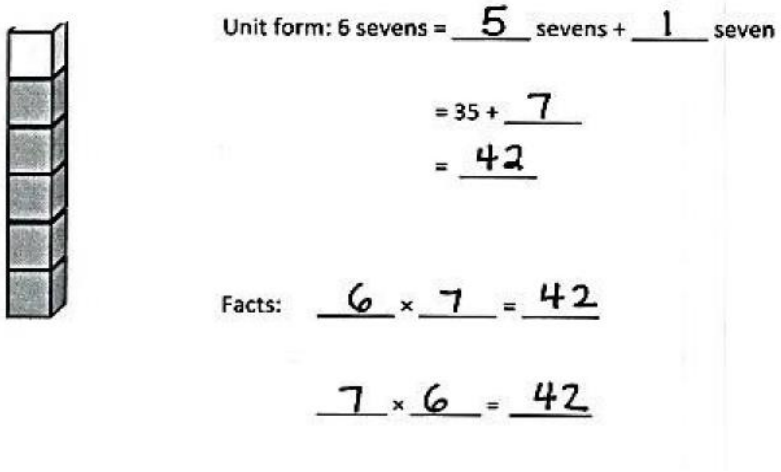
Adding to Make 10



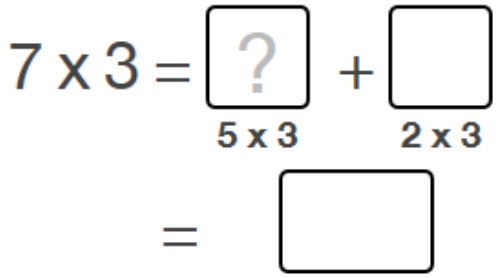
Adding to Make Hundreds



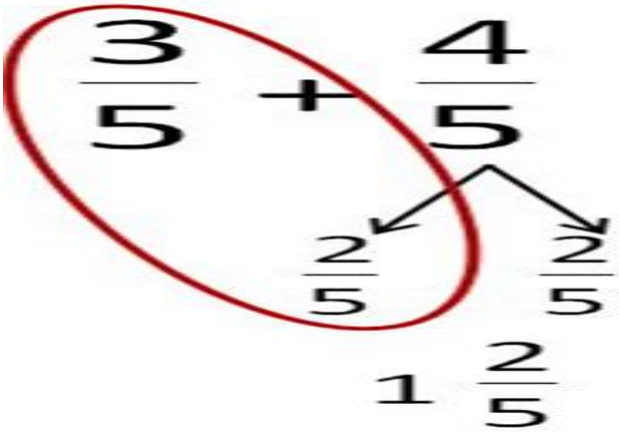
Subtraction



Multiplication (Pictoral)



Multiplication



Adding Fractions

Reflection

How can a students' lack of ability to see a number as a sum of two parts hinder their mathematical understanding?

Reflect on the practices in your district, school, and classroom. Is composition and decomposition of numbers a focus in the early grades? Why or Why not?

Making Number Relationships More Explicit

One and Two More, One and Two Less

When children count, they have no reason to reflect on the way one number is related to another. To learn that 6 and 8 are related by the twin relationships of "two more than" and "two less than" requires reflection on these ideas within tasks that promote counting. Counting on (or back) one or two counts is a useful tool in constructing these ideas.

Student Activity: ONE-LESS-THAN DOMINOES (1.OA.5)

Retrieved from www.strategicinterventionsolutions.org

Materials: Dot set dominoes

1. Students play the game in the usual way, but instead of matching ends, a new domino can be added if it is one less than the domino at the end of the board.
2. A similar game can be played for two less, one more, or two more.

Student Activity: MAKE TWO-MORE-THAN SET (1.OA.5)

Retrieved from www.strategicinterventionsolutions.org

Materials: Dot Cards

1. Provide children with about six dot cards.
2. For each card, their task is to display a set of counters that is two more than the set shown on the card.
3. Spread out eight to ten cards and allow children to form their own sets.
4. Children should read their set as "Two more than _____ is _____"

Note: You can make a set of dominoes out of cardstock and put a dot pattern on each end. The same patterns can appear on lots of dominoes with different pairs making up one set.

Templates to create paper sets of dominoes can be found here:
https://sites.google.com/site/get2mathk5/home/templates-graphic-organizers

Anchoring Numbers to 5 and 10

The knowledge of 8 as “5 and 3 more” and as “2 away from 10” can play a role in how a child thinks about these examples. Later, similar relationships can be used in the development of mental computation skills on larger numbers, such as 68 + 7.

The most common models to help children anchor numbers to 5 and 10 are five frames and ten frames. Most first and second grade classrooms will use predominantly the ten frame. For a while, many children will count every counter on their ten frame. When making a new number, some children will remove all the counters from the ten-frame and begin from a blank frame. Others will soon learn to adjust numbers by adding on or taking off only what is required, often capitalizing on a row of five without counting. DO NOT pressure children to use one approach or another. With continued practice, all children will grow. How they are using the ten-frame provides insight into children’s current number concept development.

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

Student Activity: TEN FRAME FLASH CARDS (1.OA.6)

Conklin, Melissa. *It Makes Sense! : Using Ten Frames to Build Number Sense*, 2010

1. Flash ten frame cards to the class or group and see how quickly the children can tell you how many dots are shown.
2. Variations:
    i. Saying the number of empty spaces on the card instead of the number of dots.
    ii. Saying one more than the number of dots (or two more and also less than)
    iii. Saying the “ten fact”, for example “six and four make ten”

Note: Ten frame tasks are surprisingly challenging for children. Children must reflect on the two rows of five, the spaces remaining, and how a particular number is more or less than 5 and how far away from 10.

Literature Connection: **My Little Sister Ate One Hare** by Bill Grossman
A ravenous little sister eats everything from one hare to ten peas.

1st Grade: Students can work collaboratively to find all of the different combinations the sister could eat to total 10 items.
2nd Grade: Students are given the task to find out how many critters in all did the sister eat?
Extension Activity: Ask students how many more disgusting creatures the little sister would have to eat to reach 100 or more. Have them continue with the pattern of 11 creatures, then 12, and so on.

Number Talks with Ten Frames

Another way to utilize ten frames is through number talks. By asking students to come up with a variety of ways to prove "how many" are on each ten frame, students hear various solution paths and learn additional strategies beyond just counting all.

Video: Ten Frame Number Talk

https://www.youtube.com/watch?v=PH5RG4zmmHE

https://www.youtube.com/watch?v=zdE_lxSC3V8

Note: We learned about Number Talks in a previous TDOE training. Some of the resources from that training are included in the Module 2 Appendix.

Small Group Discussion

As you watch the video clip, think about the following questions. Participants will discuss their thoughts with their small group.

- How does the teacher build student fluency with small numbers?

- What questions does the teacher pose to build understanding?

- What strategies are the students using to build meaning of the numbers?

- What opportunities are created for the students to begin building an understanding of ten?

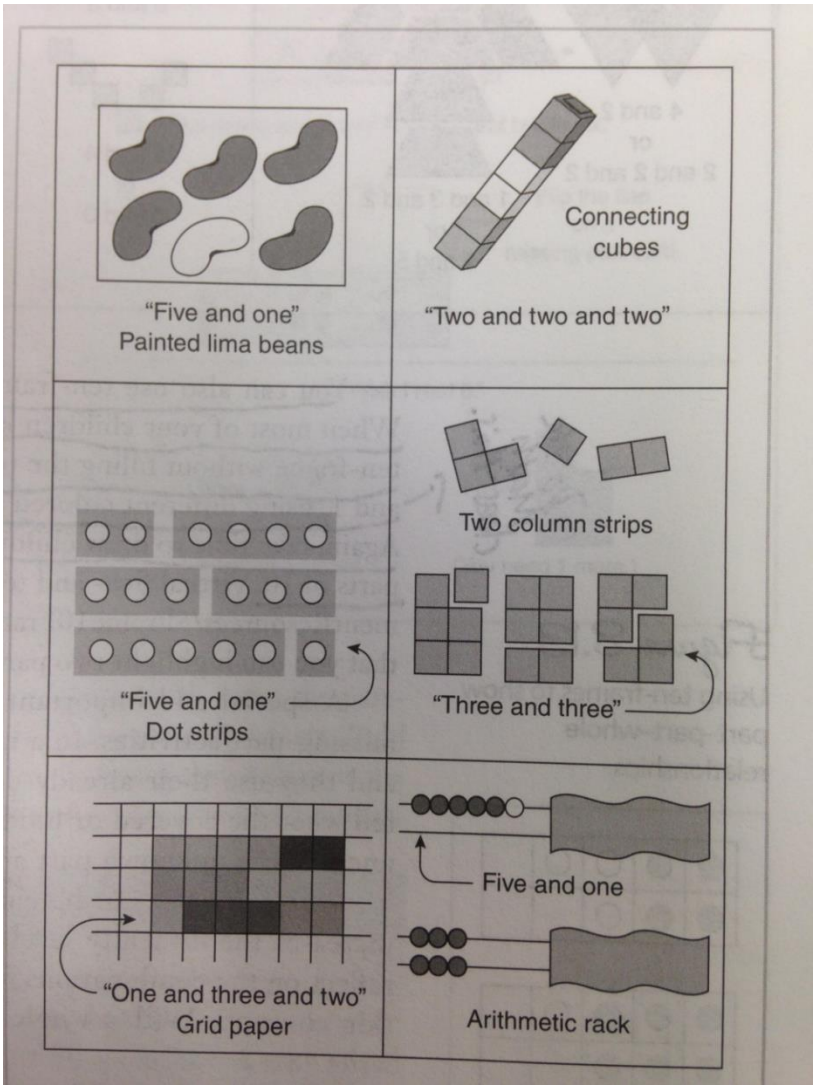
- How does the teacher support student communication during the number talk?

Part-Part-Whole Relationship

Focusing on a quantity as a sum of its parts has important implications for developing number sense. The ability to think about a number in terms of its parts is a major milestone in the development of number sense.

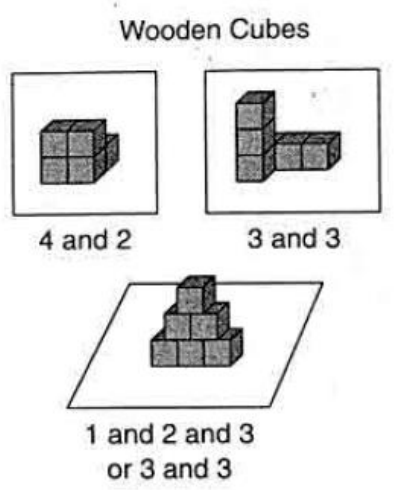
Of the four number relationships we have explored, part-part-whole ideas are easily the most important. It is not unusual to find children in the second grade who have not developed firm part-part-whole constructs for the numbers 7 through 12, even though by that time they should be adding up to 10.

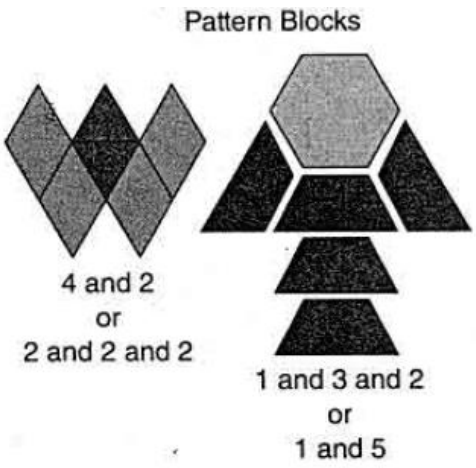
ASSORTED MATERIALS TO BUILD A NUMBER IN TWO PARTS



- Van de Walle, Lovin, Karp, & Bay-Williams. *Teaching Student Centered Mathematics*, 2014

You may add interest and vary the activity by adding a design component. Rather than creating a two-part illustration, children can create an interesting design with an assigned number of elements.





- Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Literature Connection:

MATH-terpieces: The Art of Problem Solving by Greg Tang

Greg Tang underscores the importance of four basic rules in problem-solving: Keeping an open mind, looking for unusual number combinations, using multiple skills (like subtracting to add) and looking for patterns. The book involves putting subsets of objects together to make a given sum. The following task can be used.

Ballet Shoes: Find three ways to make a 7.
Lilies: Find four ways to make an 8.
Umbrellas: Find five ways to make a 9.
Peaches: Find five ways to make a 10.
Swirling Stars: Find four ways to make a 7.
Purple Spots: Find six ways to make an 8.
Fish: Find six ways to make a 9.
Eyes: Find six ways to make a 10.
Squares: Find eight ways to make a 7.
Clocks: Find seven ways to make an 8.
Spatters: Find seven ways to make a 9.
Soup Cans: Find ten ways to make a 10.

Retrieved from http://letsreadmath.com/wp-content/uploads/2013/03/March_Teacher_Directions_2-28.pdf

Missing–Part Activities

A special and important variation of part-part-whole activities is referred to as missing-part activities. In a missing-part activity, children are given the whole amount as they use their already-developed knowledge of the parts of that whole to try to tell what the covered or hidden part is. If they do not know or are unsure, they simply uncover the unknown part and say the full combination as they would normally.

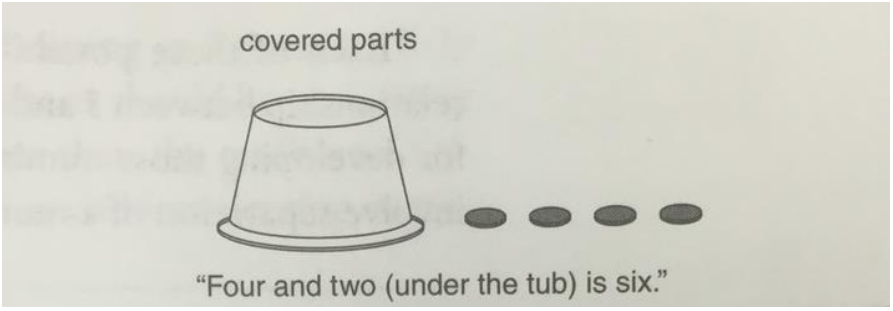
Missing-part activities can be challenging for children not only because the missing part increases the difficulty level, but also because they encourage children to continue to reflect on the combinations for the number. They also serve as a precursor to subtraction concepts. With a whole of 8 but with only 3 showing, the child can later learn to write "8-3=5."

Student Activity: COVERED PARTS (1.OA.8)

Retrieved from www.strategicinterventionsolutions.org

Materials: Counters, Cups

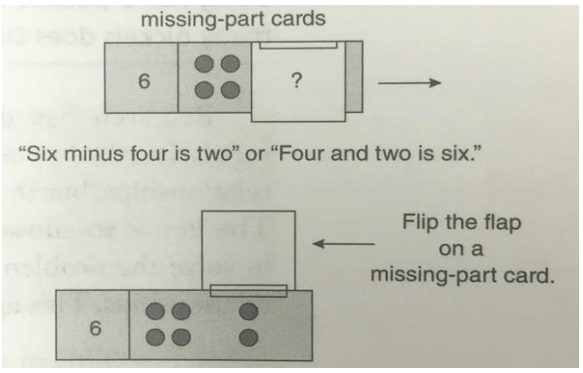
1. A set of counters equal to the target amount is counted out, and the rest is put aside.
2. One child places the counters under a solid cup and then pulls some out into view. (This amount can be none, all, or any amount in between.)
3. The partner says the two parts that makes the whole. For example, "Four and two is six."
4. If there is a hesitation or if the hidden part is unknown, the hidden part is immediately shown.



Student Activity: MISSING-PART CARDS (1.OA.8)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

1. For each number from 4 to 10, make missing part cards using strips of cardstock measuring 3 X 9 inches (this will ensure that a sticky note will fit over it).
2. Each card has a numeral for the whole and two dot sets with one set covered by a sticky note.
3. Children use the cards by saying the two parts that make a whole, such as "four and two is six".



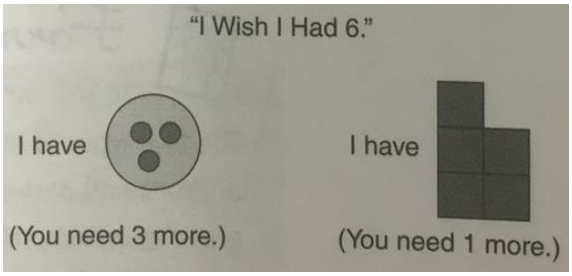
Student Activity: I WISH I HAD (1.OA.8)

Retrieved from www.strategicinterventionsolutions.org

Materials: Dot Plates, Counters

1. Hold out a dot plate, bar of connecting cubes, or other manipulative and say "I wish I had ____."
2. Students will show the amount of counters needed.

Example: Plate shows 6; "I wish I had nine";
three counters would be shown.



Student Activity: NUMBER SANDWICHES (1.OA.6)

Retrieved from www.strategicinterventionsolutions.org

Materials: Dot Cards (one set per 2 students)

Select a number between 5 and 12, and partner children to find combinations of two dot cards totaling that number.

1. Children make a "sandwich" with two cards by placing them back to back with the dot sides out.
2. When they have find at least 10 pairs, the next challenge is for the partner to name the number on the other side. The cards are turned over to confirm.
3. The same sandwiches can then be used again to name the other hidden part.

Student Activity: Sum Search

Sum Search

Find two or more addends (horizontally, vertically or diagonally) to equal the sum in the center square. Circle the addends and write the equation.

6	1	3	8	5	5	8	4	2	1
3	1	4	5	9	4	2	3	1	0
2	3	5	7	2	6	2	3	2	1
6	4	7	5	3	6	8	7	2	5
0	1	1	2	10		0	10	1	2
7	9	1	3			6	2	2	2
8	6	4	0	3	7	3	6	2	1
5	3	4	6	3	6	0	7	2	1
9	3	7	1	3	2	8	7	6	1
5	3	1	1	3	6	8	7	9	1

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Sum Search

Find two or more addends (horizontally, vertically or diagonally) to equal the sum in the center square. Circle the addends and write the equation.

5	1	3	2	5	5	8	0	2	1
3	1	4	5	6	4	1	3	1	0
2	5	5	7	2	3	2	1	2	1
6	5	7	5	3	6	8	7	2	5
0	1	1	2			0	10	1	2
7	9	1	3	9		6	2	2	2
8	6	4	0	3	7	3	6	2	1
5	3	4	1	2	1	0	6	3	5
9	3	7	1	4	2	1	7	6	1
5	2	1	1	3	6	8	7	8	1

Retrieved from http://www.k-5mathteachingresources.com/support-files/sum-search.pdf

Student Activity: BUMP (2 Player Addition Games)

Materials

- Need two dice
- Each player starts with a set of 10 same-color chips or clear counters (Player 1: 10 red Player 2: 10 yellow)

Rules:

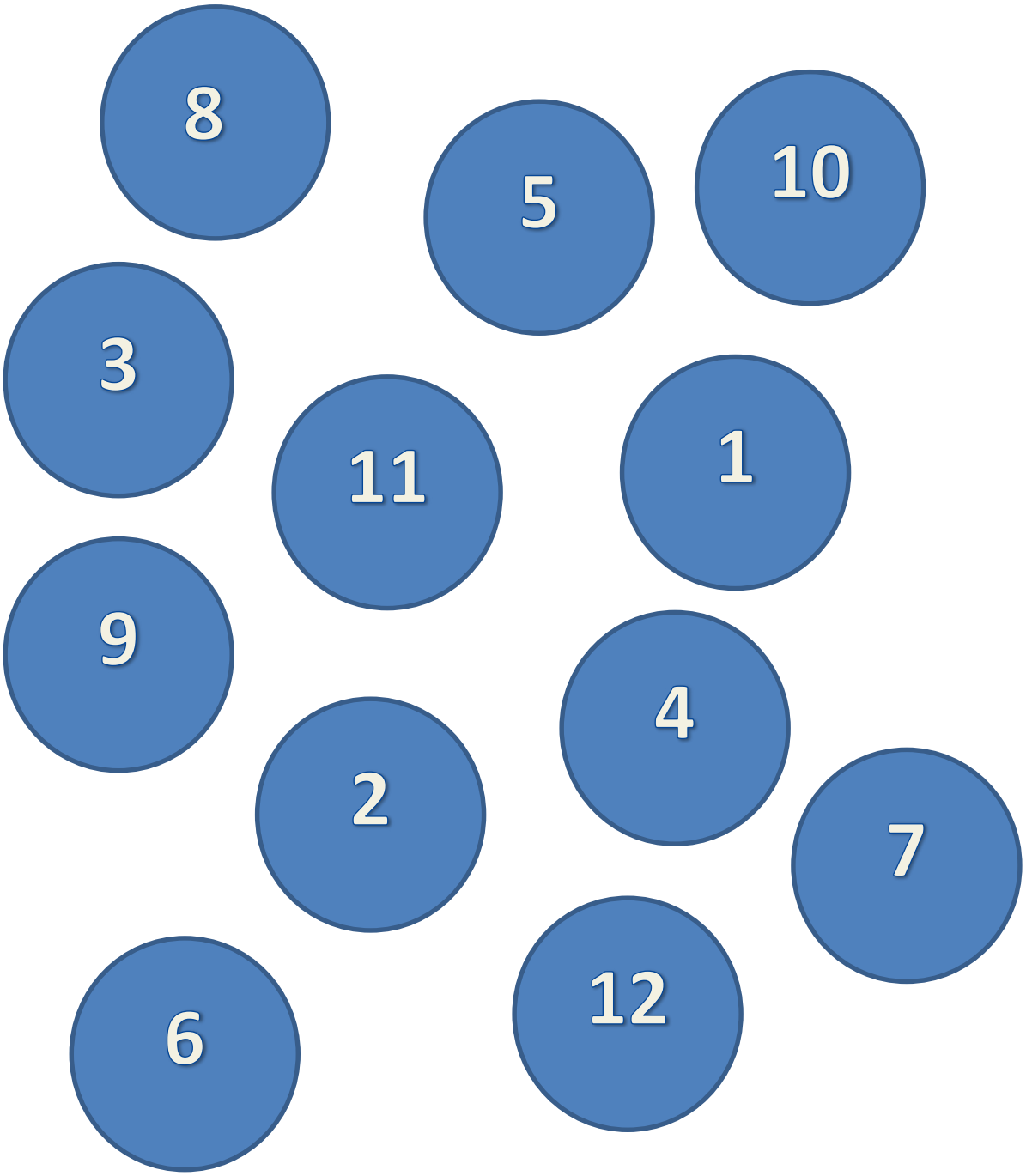
Level 1: Roll dice and put a clear counter on the circle showing the sum of the dice. If another player's counter is on that number, BUMP it off. If your marker is on that number, put an additional counter on top ("crown") and it closes that spot. The winner is the player that uses all of his/her markers first!

- If a player rolls double he gets to roll again.
- If all possible circles (sums and addends) are closed, the player loses a turn.
- The first person to get rid of all 10 chips is the winner.

Variations:

Level 2: Play the same as Level 1, however a player can either use the sum of the dice or the addend(s) of the dice (Example: a 5+4 is rolled, you can place your chip on 9 or 4 and 5).
Level 3: Play the same as Level 2, except players can use any two numbers to equal the sum of the addends (Example: if you roll 4 +5 you can use any two numbers that equal 9, such as 3 +6, 8+1, 7+2, etc.)

BUMP



-Retrieved from www.strategicinterventionsolutions.org

Participant Activity

With a partner, play BUMP using the various rules for each level.

Reflection

- What mathematical thinking does this game require?

- How do these activities help promote number sense and mathematical understanding?

Key Idea #1

Numbers are related to each other through a variety of number relationships. The number 7, for example, is three more than 4, two less than 9, composed of 3 and 4 as well as 2 and 5, is three away from 10, and can be quickly recognized in several patterned arrangements of dots. These ideas further extend to an understanding of 17, 57, and 370.

Relationships for Numbers 10 to 20

Even though pre-kindergartners to second graders experience numbers up to 20 and beyond on a daily basis, it should not be assumed that they will automatically extend the set of relationships that they have developed with smaller numbers to numbers beyond 10. Yet these numbers play a big part in many simple counting activities, in basic facts, and in much of what we do with mental computation. Relationships with these numbers are just as important as relationships involving numbers through 10. In fact, an emerging approach to number instruction is to expose children to numbers beyond 10 and even beyond 20 as soon as possible- even before formal place-value instruction.

(Fosnot & Dolk, 2001)

Although children may count by ones to count sets beyond 10, the experiences of counting and grouping help children build important initial place-value knowledge. The emphasis should be on helping children learn the number words and numerals beyond 10 rather than the traditional notions of place value. That will come as children develop strategies to add and subtract two digit numbers.

(Van De Walle, et al, 2014)

Pre- Place –Value Relationship with 10

There has been found to be a progression of three levels in children's' understanding of ten:

- *An initial concept of ten*- The child understands ten as ten ones and does not see ten as a unit. When children at this level work on a task involving tens, they will count by ones.

- *An intermediate concept of ten*- The child understands ten as a unit composed of ten ones but relies on materials or representations to help complete tasks involving tens.

- *A facile concept of ten*- The child can solve tasks involving tens and ones without using materials or representations. At this level children can mentally think about two- digit numbers as groups of tens and ones.

(Wright et al., 2006)

Stop and think: Say to yourself, "One ten." Now think about that from the perspective of a child just learning to count to 20! What could one ten possibly mean when ten tells me who many fingers I have and is the number that comes after nine? How can it be one? How would you explain this to a child?

Formal Assessment Note:

Ask a child to count out 35 tiles. Watch closely to note whether they count out the tiles one at a time and put them into a pile without any type of grouping or if they group them into tens. Have the child write the number that tells how many tiles they just counted. Some may write "53" instead of "35", a simple reversal. You will likely find that early on children count the tiles one by one and are not yet thinking of ten as a unit (level 1 or initial concept of ten).

In order to help children begin thinking about counting in ways that can advance their understanding of ten, consider providing lots of purposeful opportunities for them to count and group objects.

Participant Activity: Make a set of Place Value (Hide Zero) Cards

Place Value or Hide Zero Cards is a very effective mathematical tool that can be used in a variety of manners. In the module appendix, find and cut out your own set of Place Value cards. We will use these throughout the modules. It is necessary when using them in your classroom that each student has their own set.

Student Activity: BUILD THE NUMBER with HIDE ZERO CARDS (1.NBT.2, 2.NBT.1, and 2.NBT.3)

Hazekamp, Jana. *Why Before How: Singapore Math Computation Strategies*, 2011

1. From cardstock create a set of cards. The tens card should be twice as long as the ones card, and the hundreds cards three times as long as the ones card.

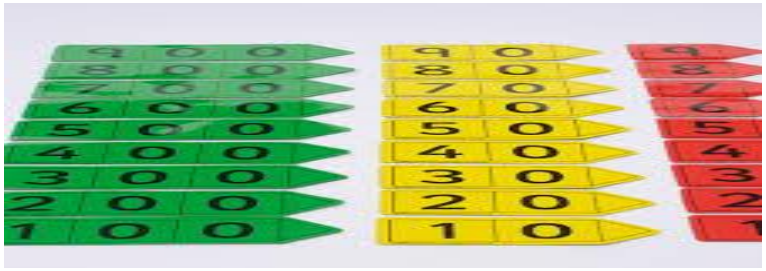
2. Children use the cards to create two-digit numbers.

3. Write a two digit number where children can see it, example 26.

4. Children are to find the two cards that can be used to make 26 (20 and 6).

5. Select children to demonstrate to the class how to make the given number. **Point out that you can still see the 20 hiding under the overlay of the 6.**

6. Repeat the activity with different two-digit numbers. When children are ready you can extend the activity to three-digit numbers. You may also want children to model the numbers with other materials such as base ten materials or ten frames.



Student Activity: TEN AND SOME MORE (1.NBT.2)

Van de Walle, Lovin, Karp, & Bay-Williams., 2006

Materials: Two Part Mat; up to 19 counters per child

1. Using a simple two-part mat, have children count out ten counters onto one side.
2. Have them put five counters on the other side.
3. Together count all of the counters by ones.
4. All together say the combination, "ten and five is fifteen."
5. Turn the mat around, "five and ten is fifteen."
6. Repeat with other numbers in a random order but without changing the side of the mat.

This activity is designed to teach new number names and therefore does require a certain amount of direct teaching. Following this activity, explore numbers to 20 in a more open-ended manner. You may provide children with a two-part mat that has two ten frames drawn one under the other. In random order, have children show numbers to 20 on their mats. There is no preferred way to do this as long as the number of counters is correct.

Formal Assessment Note:

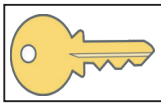
It is interesting to discuss how the counters can be placed on the mat so that it is easy to see how many are there. Have children share their ideas. Not every child will use a full set of ten, but as this idea becomes more popular the notion that 10 and some more is a teen amount will soon develop. Don't forget to include numbers less than ten as well, and if your students are up for the challenge you may include larger numbers, such as 27.

Student Activity: MORE AND LESS EXTENDED (1.NBT.5 and 2.NBT.8)

Conklin, M., 2010

Materials: projector, counters, filled ten frame

1. On a projector, show seven counters and ask what is two more, two less, one less, and so on.
2. Now add a filled ten frame to the display (or ten in any pattern) and repeat the question.
3. Pair up questions by covering and uncovering the ten frame.
4. This activity can be extended to larger numbers.



Key Idea #2: Having number sense means that you can think about and use numbers and their relationships in many different ways.

Numbers to 100: Early Introductions

According to the Tennessee Academic Standards, kindergartners are expected to be able to count to 100 by the end of the school year. Therefore, early exposure to numbers to 100 is important even in kindergarten. Although it is extremely unlikely that children in kindergarten or first grade will have a true understanding of tens and ones or place value, they can learn much about the sequence of numbers to 100, and beyond. Most important at this early level is for children to become familiar with the counting patterns to 100.

The hundreds chart is an essential tool for every K-2 classroom.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Participant/Student Activity: PATTERNS ON A HUNDREDS CHART

Conklin, M & Sheffield S. *It Makes Sense! : Using the Hundreds Chart to Build Number Sense*, 2010

Directions:

1. Work in pairs to find patterns on the hundreds chart.
2. Solicit ideas orally from the class.
3. Have children explain patterns found by others to be sure that all understand the ideas.

Small Group Discussion

- What patterns do you focus on with your students?
- What other patterns do you see that may support Number Sense, Place Value, or Addition and Subtraction Concepts?

Patterns on a Hundred Chart

Module 2 Closing Activity

Think back to the areas we have explored concerning Number Sense Concepts:

- Spatial Relationships
- One More/Less, Two More/Less
- Anchoring to Numbers 5 and 10
- Part-Part-Whole Relationships
- Relationships to Numbers 10 to 20
- Numbers to 100

Note: Although we have looked at a variety of activities, this is not an exhaustive list.

Reflection

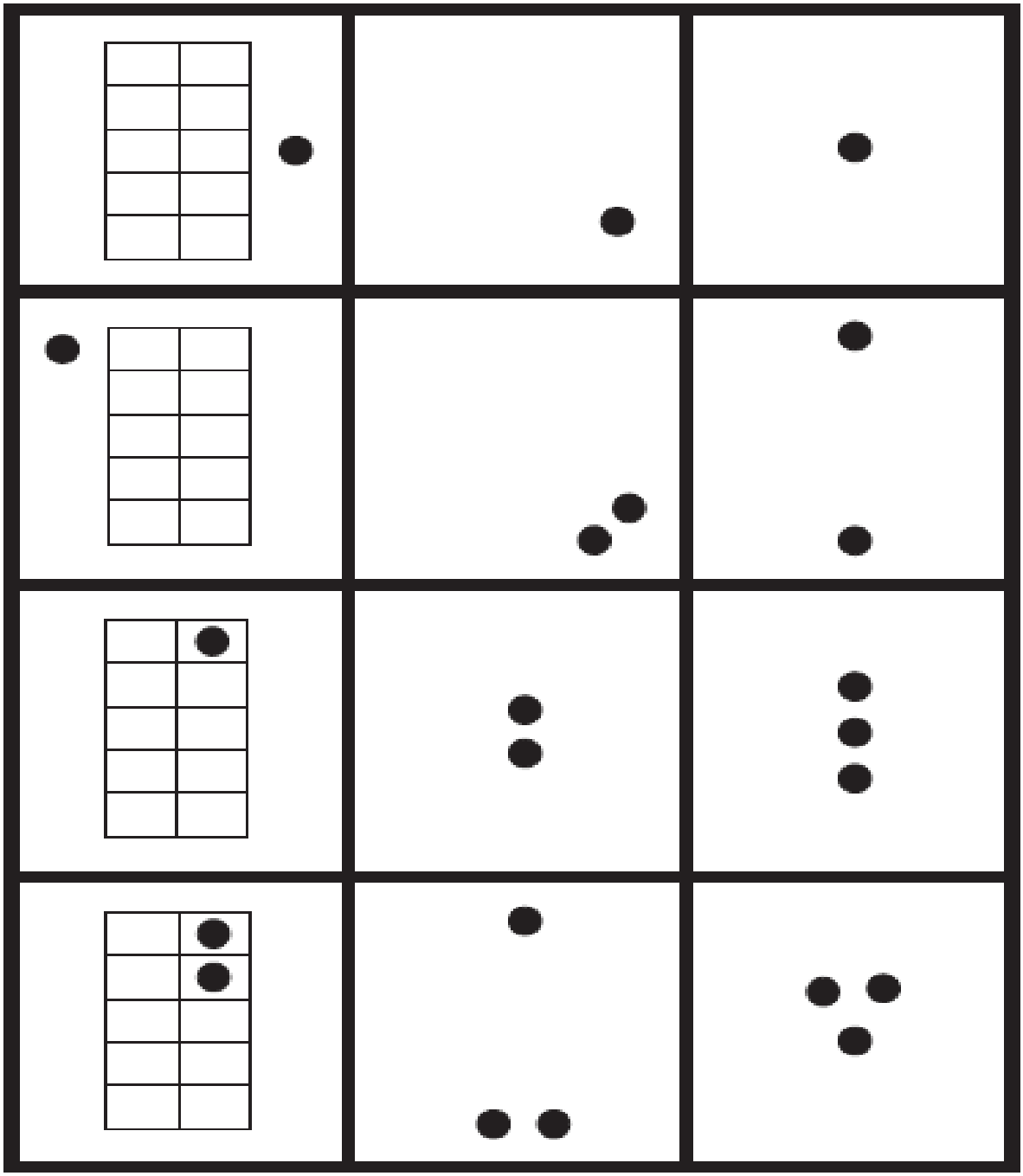
- What other strategies/activities do you use in your classroom to promote Number Sense Concepts?

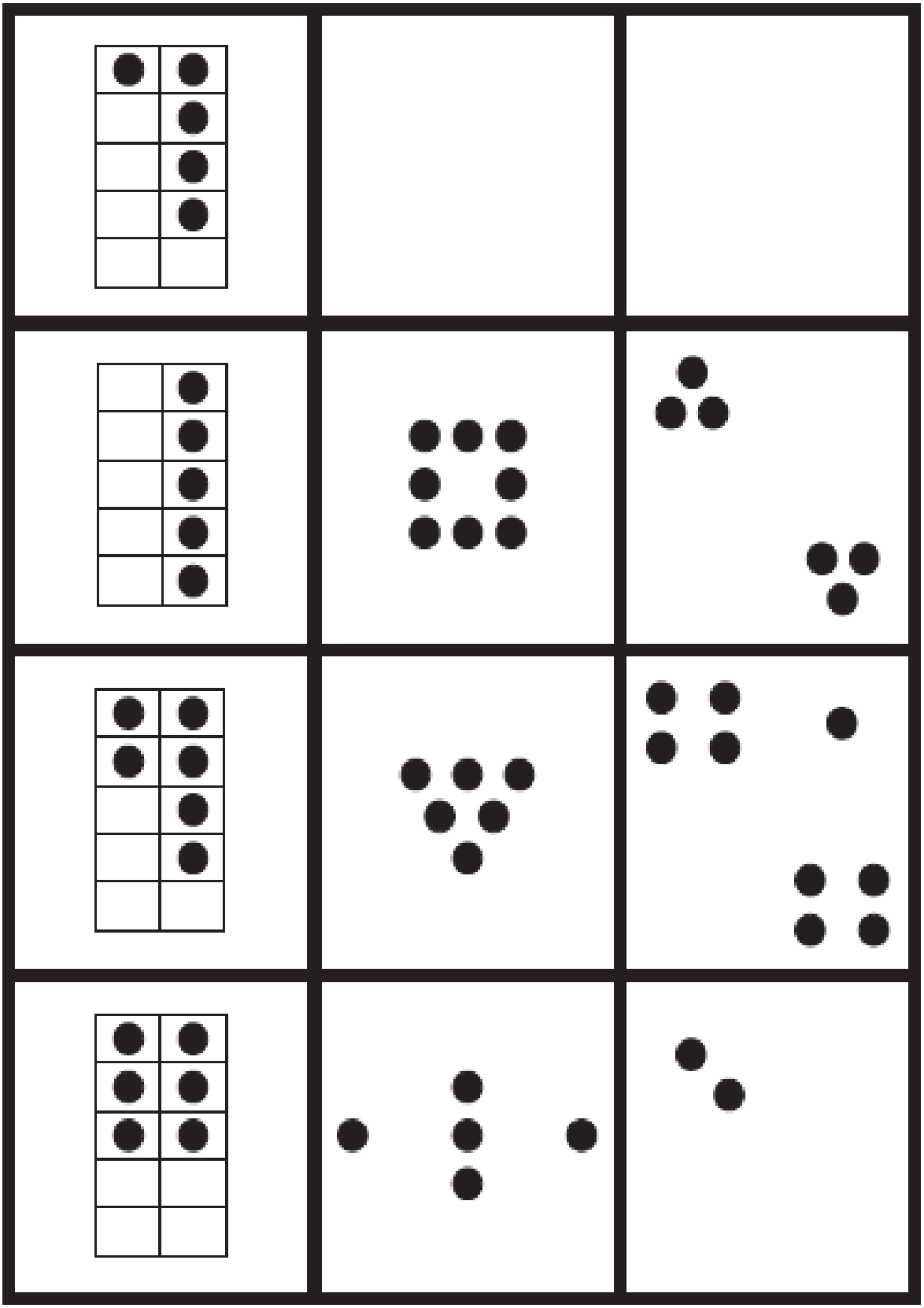
- Discuss these activities/strategies in your small group. Be prepared to share whole group. Record any ideas that you would like to incorporate in your classroom in the following pages.

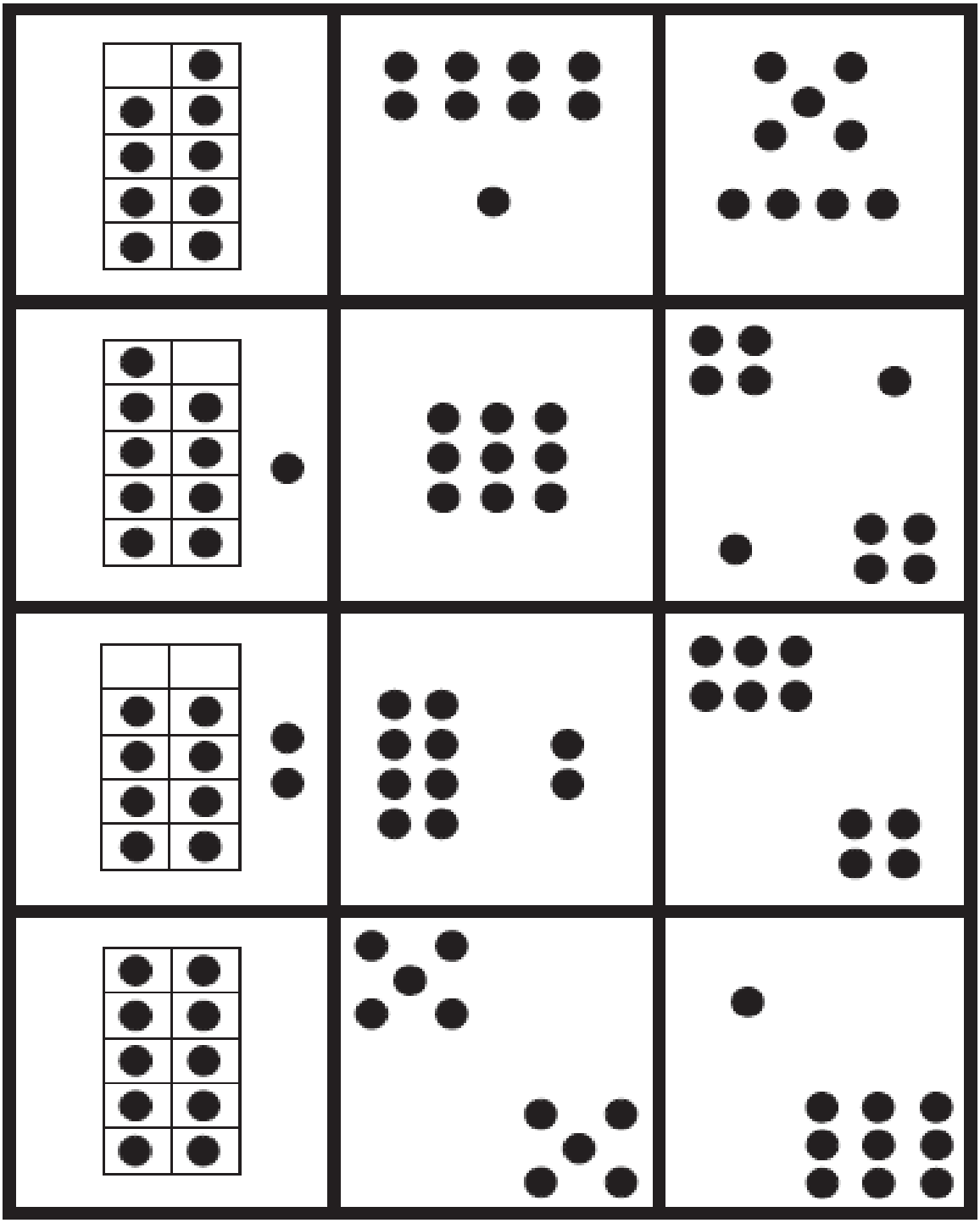
Collaborative Notes/Activities – Number Sense

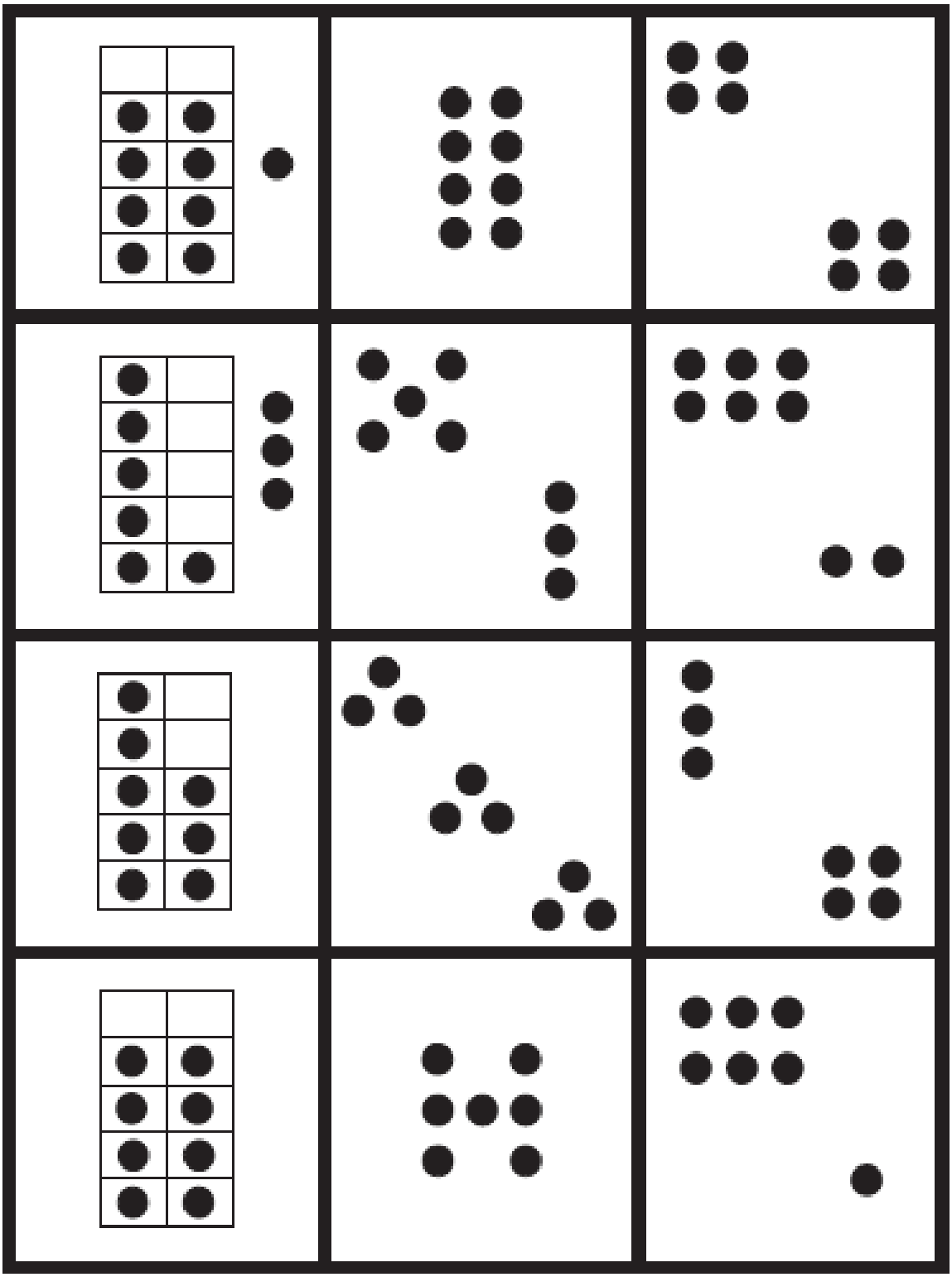
Collaborative Notes/Activities – Number Sense

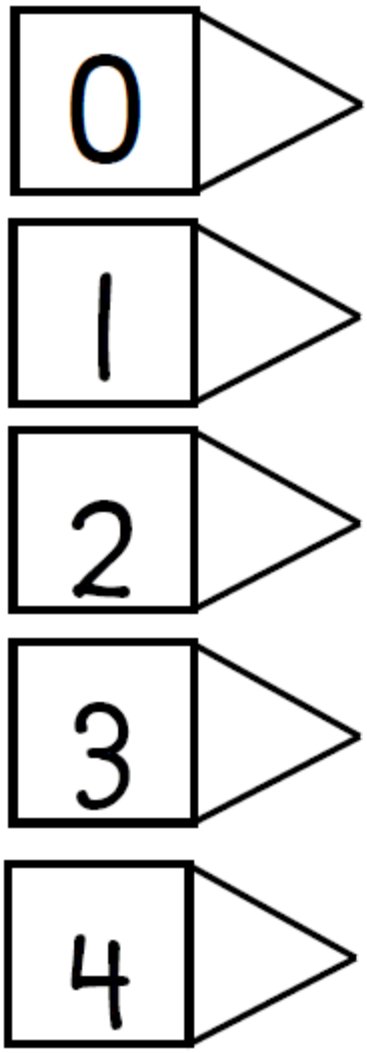
Module 2 Appendix







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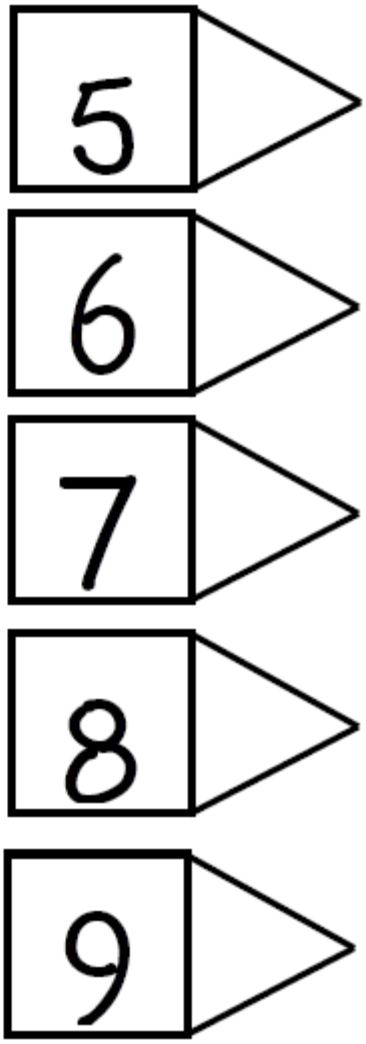
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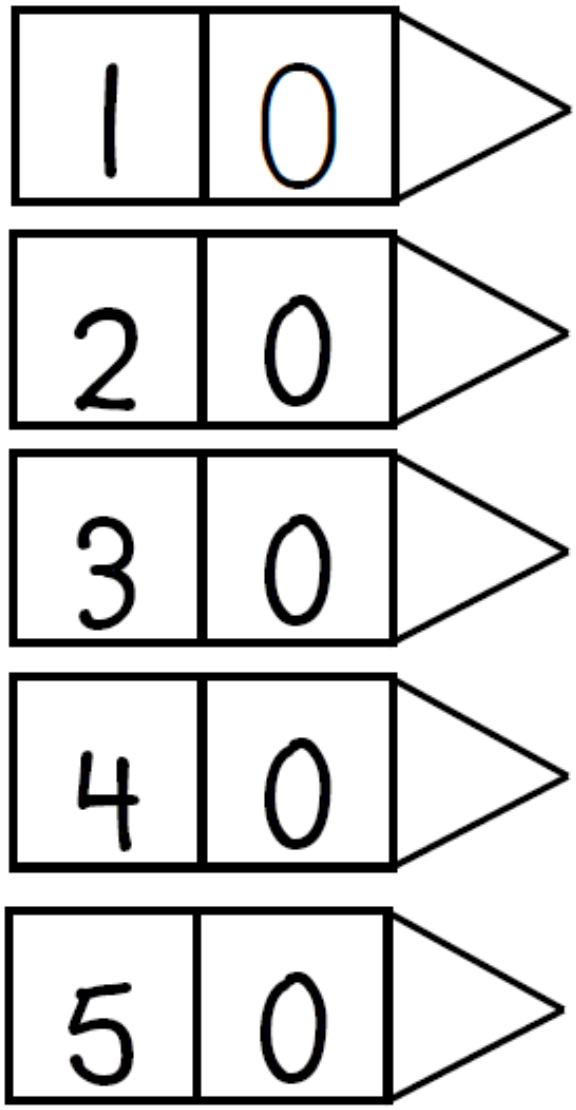
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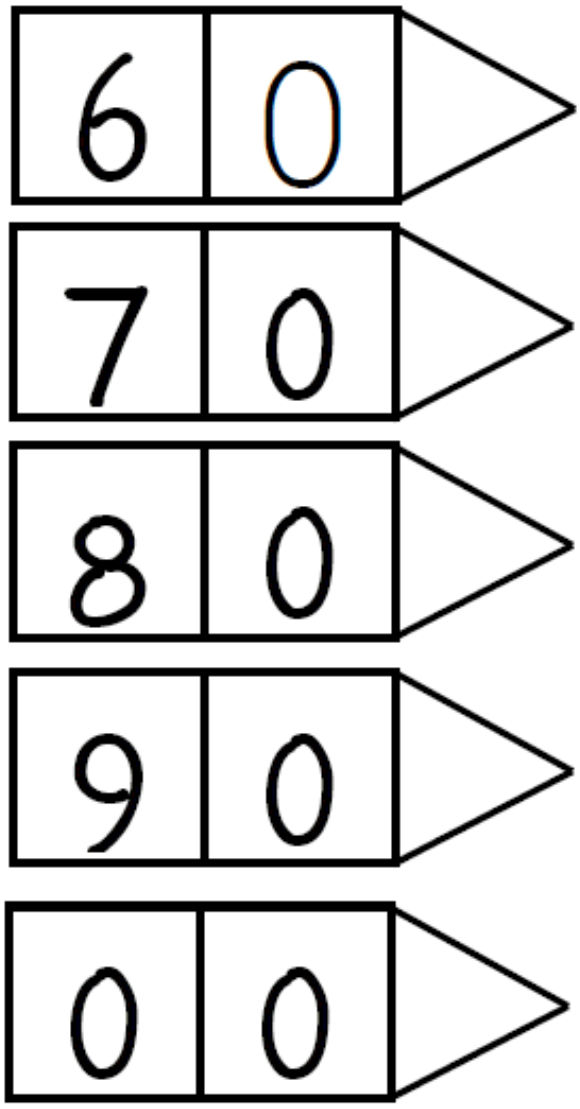
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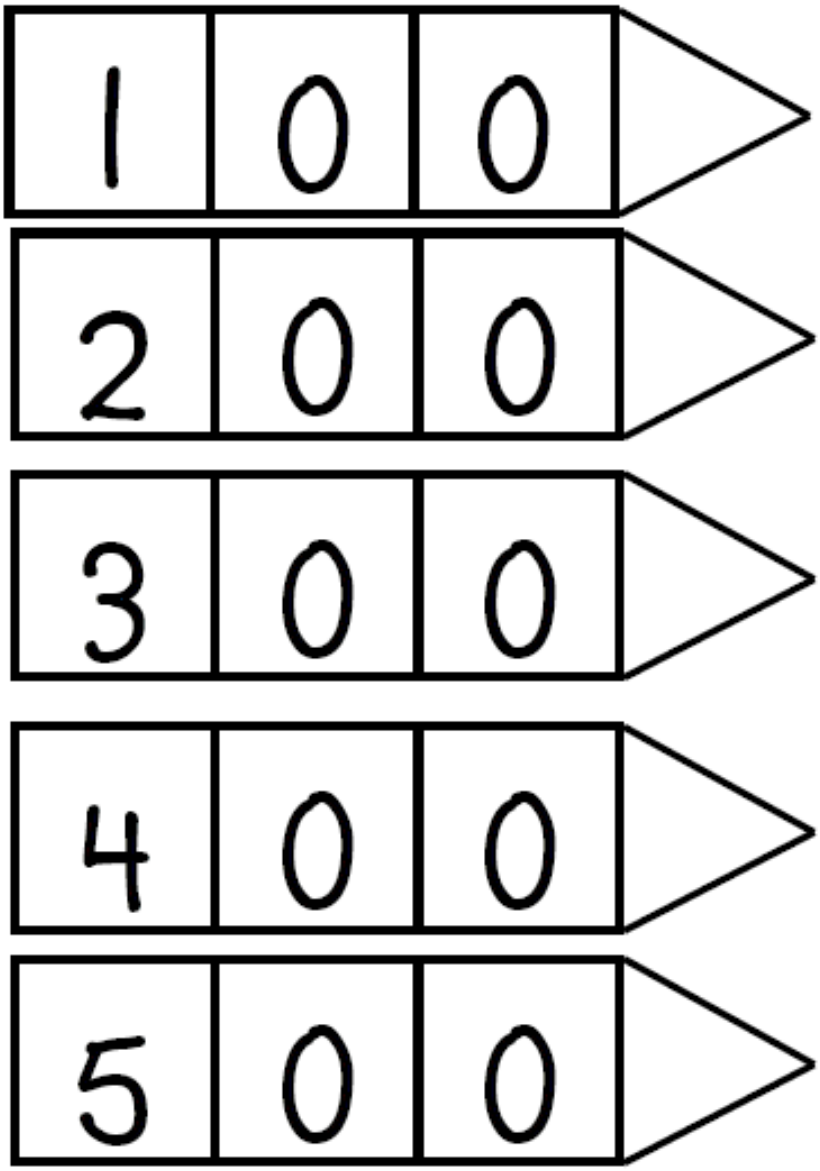
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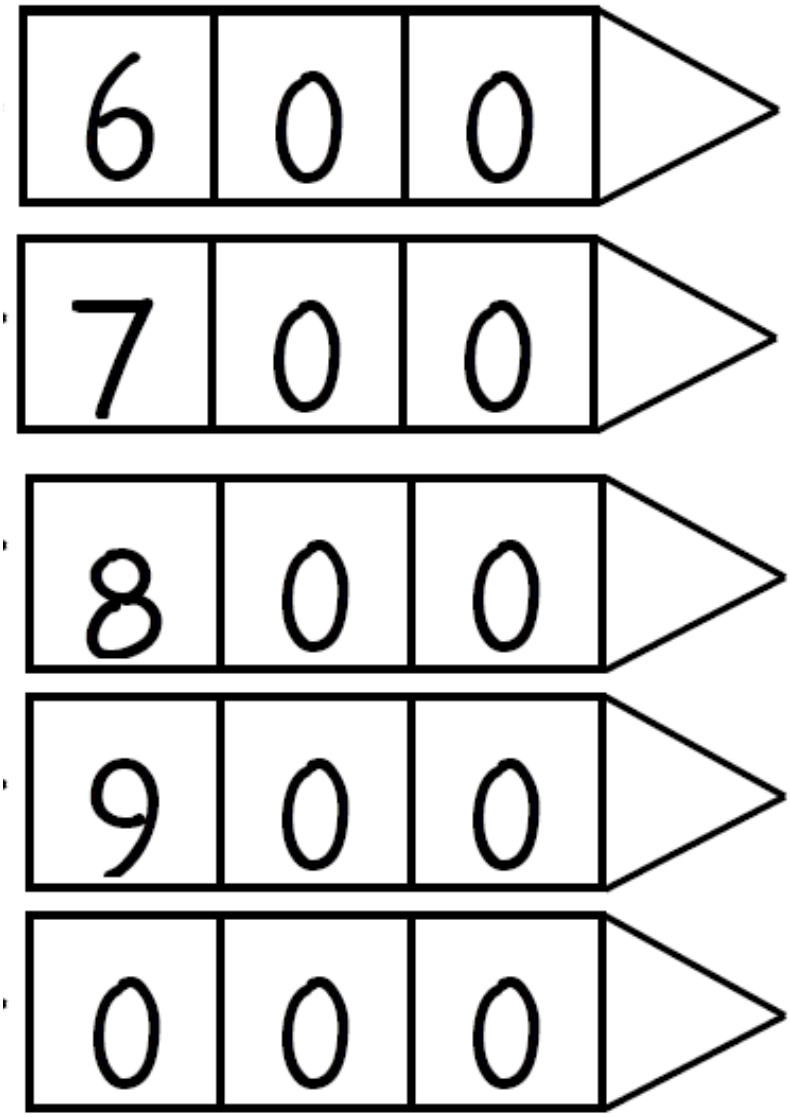
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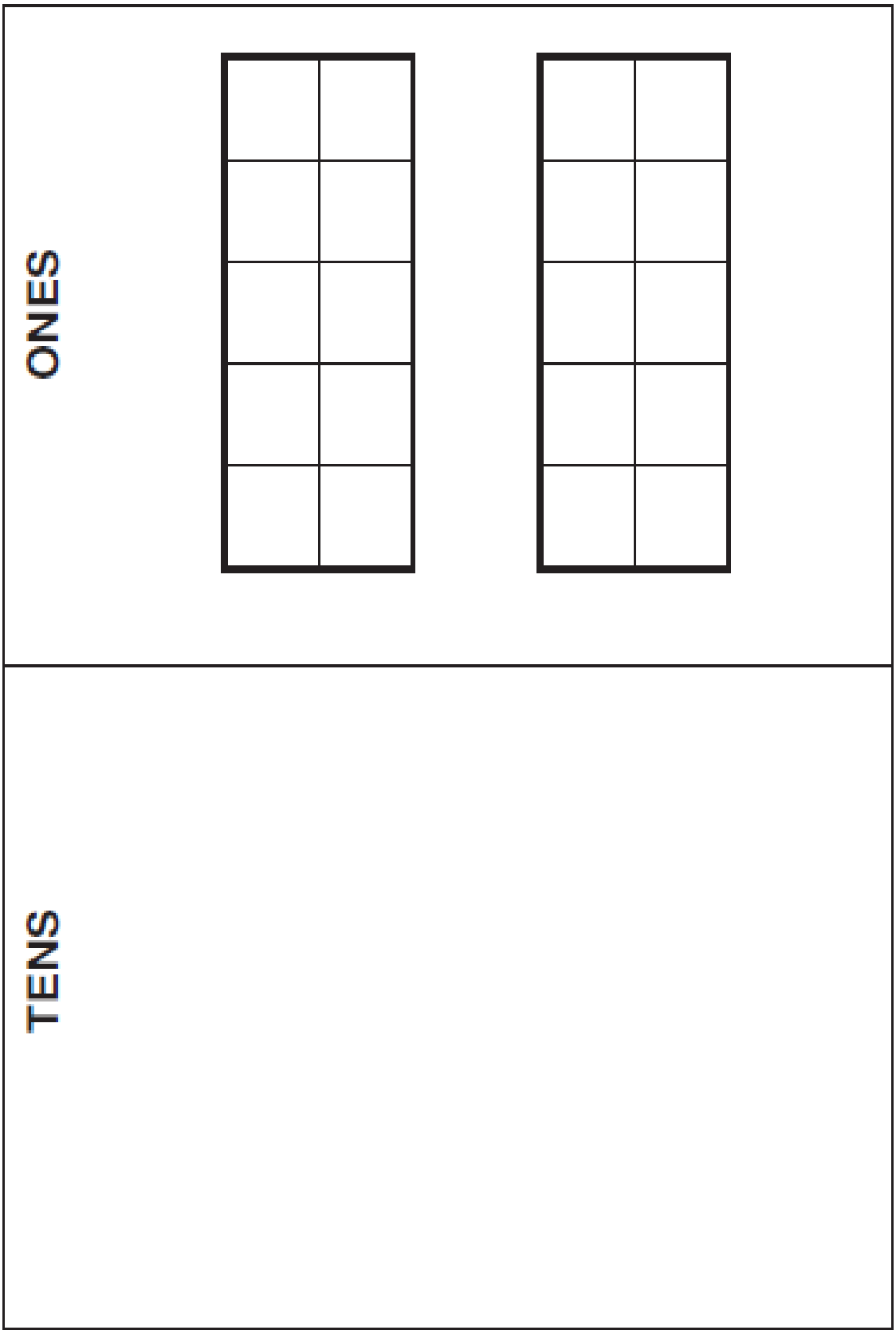
ONES
TENS

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Number Talks

- Number talks are classroom conversations and discussions around purposefully-crafted computation problems. They are opportunities for students to share mathematical thinking.
- Number talks are designed to elicit specific strategies that focus on number relationships and number theory.
- Computational Goals: Accuracy, Flexibility, and Efficiency.

Number Talks are...	Number Talks are not...
A concentrated period of 5 to 15 minutes --3 to 5 times per week --whole group or small group	An entire math lesson
Purposefully-crafted computation problems that students are expected to solve mentally with accuracy, efficiency, and flexibility	Unrelated and disjointed problems --calendar time --number of the day
Rooted in classroom discussions --errors are used as opportunities to unearth misconceptions	Pre-teaching strategies

- Parrish, 2014.

Number Talk Tips

- During pre-planning of number talks, anticipate possible strategies that might arise with specific problems.
- If you aren't able to follow a student's strategy, it's okay to tell them you want time to think about their strategy more deeply.
- Make sure your notation is mathematically correct as you record student thinking.
- Let students start with partner talk until they get used to sharing whole group.
- Have students complete an exit problem requiring them to solve the problem two ways and to place an asterisk by the most efficient strategy.

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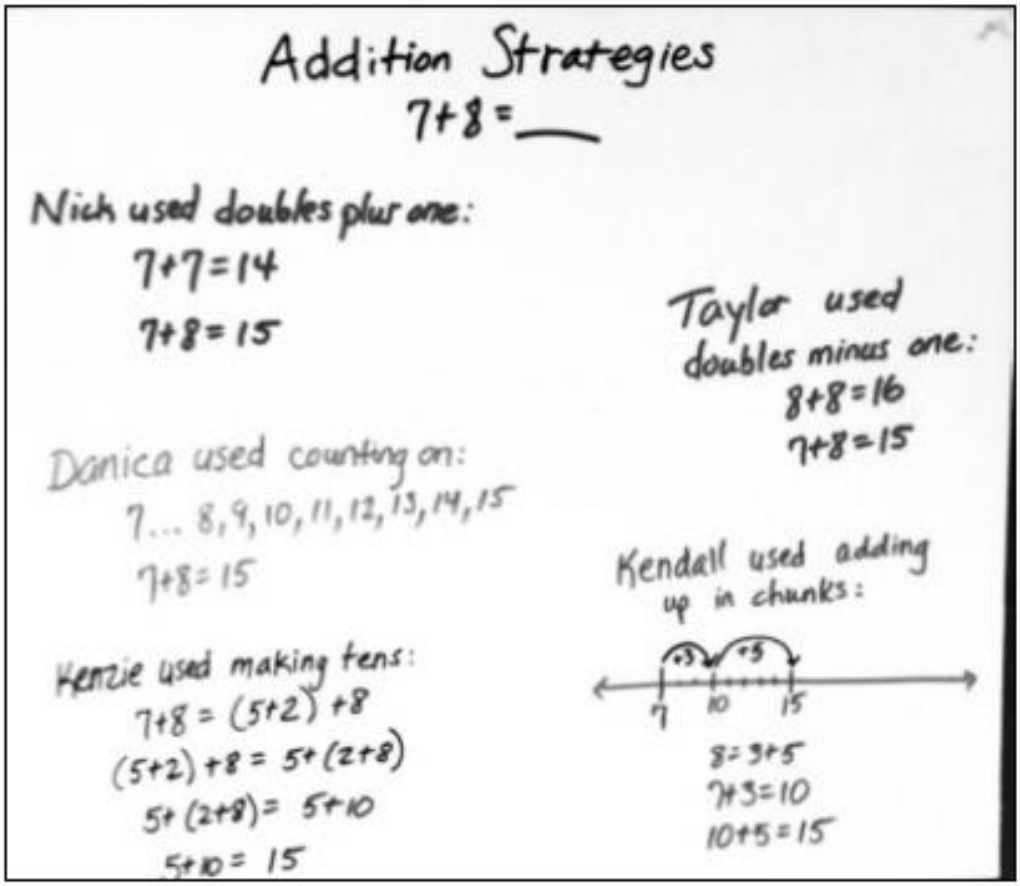
Teacher's Role in Number Talks

Your Goal:
Provide a clear path for students to see where every number in suggested solutions comes from.

How?
Purposeful Recording.

Ask Questions:
Push and question to get to what students are thinking.

Create and Post Class Strategy Charts:
Similar to anchor charts in reading, class strategy charts provide a reference for students as they work.



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First Grade Number Talks

The number talk examples are designed to provide first graders with opportunities to continue to develop fluency with numbers up to ten.

Each grouping is designed to be accomplished in a single session of 5-15 minutes. The sequence of problems allows students to apply strategies from previous problems to subsequent problems. Although students may solve the computation problems with multiple reasoning strategies, the "number talks" are grouped according to the strategy that is intended to be fostered.

Addition: Counting All/Counting On

3 + 6
3 + 7
3 + 8

9 + 1
9 + 3
9 + 5
9 + 7

Addition: Doubles/Near Doubles

4 + 4
4 + 3
3 + 3
3 + 4

8 + 8
8 + 9
9 + 9
9 + 10

15 + 15
15 + 16
14 + 14
14 + 15

Addition: Make Ten

5 + 5
5 + 5 + 4
5 + 3 + 5

1 + 8 + 9
9 + 3 + 1
1 + 6 + 9

Addition: Making Landmark or Friendly Numbers

8 + 2
8 + 2 + 4
8 + 6
8 + 5

20 + 5
19 + 1 + 4
19 + 5
19 + 8

Parrish, S. (2014). *Number Talks: Helping Children Build Mental Math and Computation Strategies.* Sausalito: Math Solutions.

Second Grade Number Talks

The number talk examples are designed to provide second graders with opportunities to continue to develop fluency with numbers using addition and subtraction reasoning strategies.

Each grouping is designed to be accomplished in a single session of 5-15 minutes. The sequence of problems allows students to apply strategies from previous problems to subsequent problems. Although students may solve the computation problems with multiple reasoning strategies, the "number talks" are grouped according to the strategy that is intended to be fostered.

There are three categories of number talks under each reasoning strategy. Category 1 represents introductory number talks designed to foster the use of a specific strategy. Category 2 number talks are to be used with students who are successfully using the specified strategy. Category 3 number talks allow students to have opportunities to extend their use of the targeted strategy.

Addition: Doubles/Near Doubles

Category 1:	Category 2:	Category 3:
6 + 6	11 + 11	25 + 25
6 + 5	12 + 12	24 + 25
6 + 7	11 + 12	25 + 26
	11 + 10	24 + 26

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Addition: Make Ten

Category 1:	Category 2:	Category 3:
2 + 5 + 8	4 + 6 + 8 + 2	8 + 2
4 + 7 + 6	9 + 3 + 1 + 7	8 + 5
5 + 5 + 8	5 + 6 + 5 + 4	8 + 4
		8 + 7

Addition: Making Landmark or Friendly Numbers

Category 1:	Category 2:	Category 3:
20 + 5	30 + 5	30 + 50
19 + 5	28 + 2 + 3	29 + 49
19 + 7	28 + 5	28 + 48
19 + 8	28 + 7	29 + 48
	28 + 16	

Addition: Breaking Each Number into Its Place Value

Category 1:	Category 2:	Category 3:
20 + 20	13 + 18	38 + 58
23 + 25	16 + 15	67 + 17
24 + 21	17 + 14	44 + 38
22 + 26	12 + 19	25 + 66

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Addition: Compensation

Category 1:	Category 2:	Category 3:
14 + 9	7 + 19	17 + 23
9 + 7	5 + 29	22 + 28
15 + 9	39 + 8	35 + 27
19 + 6	49 + 6	38 + 36

Addition: Adding Up in Chunks

Category 1:	Category 2:	Category 3:
23 + 10	26 + 10	29 + 10
23 + 20	26 + 30	29 + 15
23 + 40	26 + 50	29 + 20
23 + 50	26 + 53	29 + 24

Subtraction: Think Addition (Adding Up)

Category 1:	Category 2:	Category 3:
20 – 15	23 – 19	50 – 24
20 – 14	23 – 16	50 – 39
20 – 12	23 – 14	56 – 28
20 – 11	23 – 9	56 – 17

Subtraction: Removal

Category 1:	Category 2:	Category 3:
12 – 2	47 – 10	54 – 10
12 – 5	47 – 16	54 – 18
15 – 5	47 – 20	52 – 30
15 – 6	47 – 24	52 – 34

Parrish, S. (2014). *Number Talks: Helping Children Build Mental Math and Computation Strategies.* Sausalito: Math Solutions.

Module 3: Place Value and Base Ten

Module 3: Place Value and Base Ten

Objective

- To understand how children progress through the levels of understanding regarding "ten"

Tennessee Standards

1.NBT 2 Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a "ten."
b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

1.NBT.5 Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.

1.NBT.6 Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

2.NBT.1 Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens — called a "hundred."
b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2.NBT.3 Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2..., and represent whole-number sums and differences within 100 on a number line diagram.

TEAM Alignment

- Standards and Objectives
- Presenting Instructional Content
- Activities and Materials
- Questioning
- Teacher Content Knowledge
- Thinking
- Problem Solving

Participant Activities:

- Participants will discuss the foundational ideas of place value and consider the oral language involved.
- Participants will reflect on the different models used for developing place value understanding.
- Participants will engage in and review multiple student tasks for developing place value understanding.

Place Value Development

"A complete understanding of place value, including extensions to decimal numeration, develops over the elementary and middle grades. However, the most critical period in this development occurs in grades PreK–2. In kindergarten and 1st grade, children count and are exposed to patterns in the numbers to 100. Most importantly they begin to think about groups of ten objects as a unit. The (Tennessee State) Standards recommends that kindergarteners work with numbers between 11 and 19 and by composing and decomposing them into tens and ones using materials and drawings. By second grade, these initial ideas are extended to three-digit numbers. As a significant part of this development, children should engage in composing and decomposing numbers in a wide variety of ways as they solve addition and subtraction problems with two- and three-digit numbers. In other words, there is no need to separate place-value instruction from computation instruction. Children's efforts with the invention of their own computation strategies will both enhance their understanding of place value and provide a firm foundation for flexible methods of computation."

John Van De Walle, et al (2014)

Place Value Warm-Up Task

Use the Place Value Cards we made yesterday to complete this activity.

1. What number is 10 less than 56?
2. What number is 10 more than 89?
3. What number is 30 more than 42?
4. What number is 40 less than 46?
5. What number is 10 more than 963?
6. What number is 100 less than 809?

Note: This task acts as a bridge between understanding place value and using strategies based on place value for addition and subtraction.

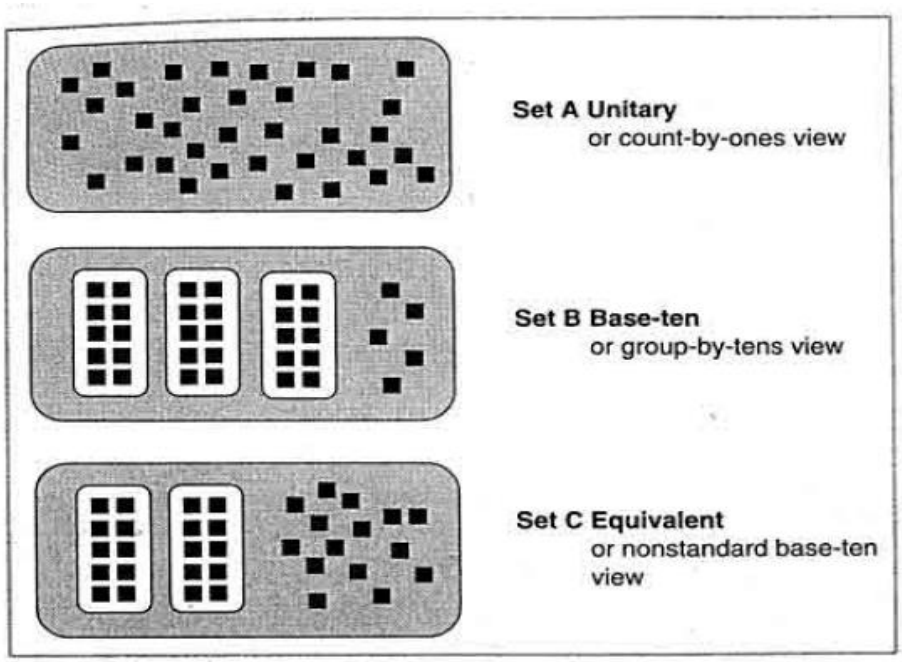
Adapted from
https://www.illustrativemathematics.org/content-standards/2/NBT/A/1/tasks/94

Foundational Ideas in Place Value

Place value understanding requires an integration of new and sometimes difficult-to-construct concepts of grouping by tens (the base ten concept) with procedural knowledge of how groups are recorded in our place value system and how numbers are written and spoken.

Integration of Base Ten Groupings with Counting by Ones

Once you recognize that children can count out a set of 35 by ones, you want to help them see that making groupings of tens and leftovers is a way of counting the same quantity.



- Van de Walle, Lovin, Karp, & Bay-Williams., 2014

You want children to construct the idea that all of these are the same and that the sameness is evident by virtue of the groupings of tens. There is a subtle yet profound difference between children at this stage. The children in the pre-base ten stage may not be sure how many they will get if they count the tiles in set B by ones, or if the groups were "ungrouped " how many there would be.

Set C – Groupings with fewer than the maximum number of tens are referred to as equivalent groupings or equivalent representations.

Understanding the equivalence of sets B and C indicates that grouping by ten is not just a rule that is followed, but that any grouping by tens, including all or some of the singles, can help tell how many. Many computational techniques (i.e., regrouping in addition and subtraction) are based on equivalent representations of numbers.

The Role of Counting in Constructing Base Ten Ideas

Counting plays a key role in constructing base ten ideas about quantity and in connecting these concepts to symbols and oral names for numbers. Each approach helps children think about mathematics in a different way.

1. *Counting by ones*- Before base ten ideas develop, counting by ones is the only approach by which children can be convinced that all three sets are the same amount.
2. *Counting by groups and singles*- (Set B) This may sound like " One, two, three groups of 10, and one, two, three, four, five singles." This method may be very novel for a student that has never thought about counting a group of objects as a single item. Notice this counting does not tell directly how many items there are. This counting must be coordinated with counting by ones before it can be a means of telling "how many".
3. *Counting by tens and ones*- (Set B and C) This is the way adults would typically count. Although this count ends by saying the number of objects it is not as explicit as the second method in counting the groups.

Reflection

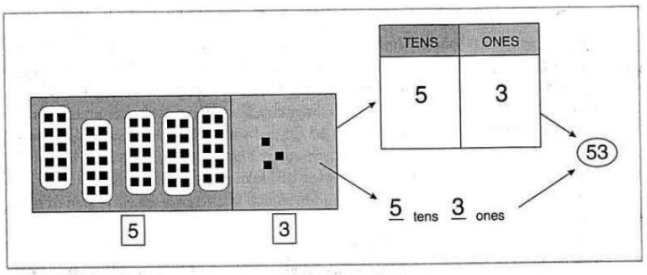
- What are some defining characteristics of children with pre-place value understanding (Module 2) and children who understand place value?

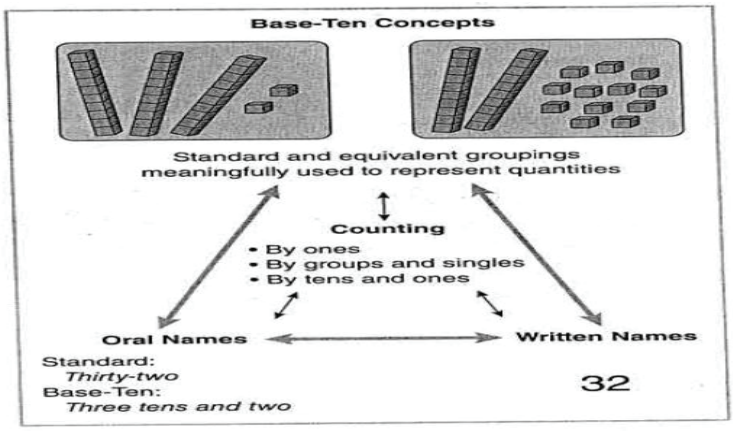
Integration of Grouping with Words

The way we say a number such as “thirty-five” must also be connected with the groupings by ten concept. There are several variations of base ten language for 35: “3 tens and a 5”, “3 tens and 5 ones”, “3 groups of 10 and 5 ones”, “ 3 tens and 5 singles” and so on. Each may be used interchangeably with the standard name. If you have ESL or ELL students, it is best to select one variation and consistently connect it to the standard language.

Integration of Groupings with Place Value Notation

The symbolic scheme that we use for writing numbers (ones on the right, tens to the left of the ones, and so on) must be coordinated with the grouping scheme. Activities can be designed so that children physically associate groupings of tens and ones with the correct recording of the individual digits.





Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Reflection

- What do you notice your students have difficulty with when interpreting and using base ten language?

Base-ten Models for Place Value

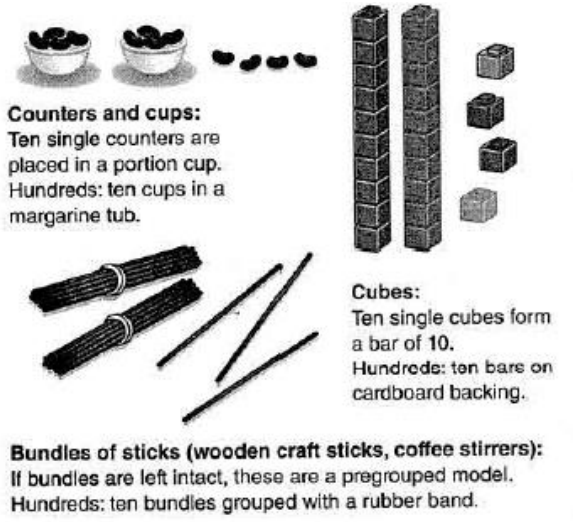
Physical models for base ten concepts can play a key role in helping children develop the idea of “a ten” as both single entity and as a set of 10 units.

Remember, though, that physical models do not “show” the concept to the children. The children must mentally construct the “ten makes one” relationship and impose it on the model. When first teaching place value, the base-ten model for ones, tens and hundreds should be proportional. That is a model for the ten is physically ten times larger than the model for a one, etc.

-Van De Walle, Lovin, Karp, & Bay Williams, 2014

Groupable Models

Models that most clearly reflect the relationship of ones, tens, and hundreds are those for which the ten can actually be made and grouped from single pieces. These could also be called “put-together-take-apart “models.

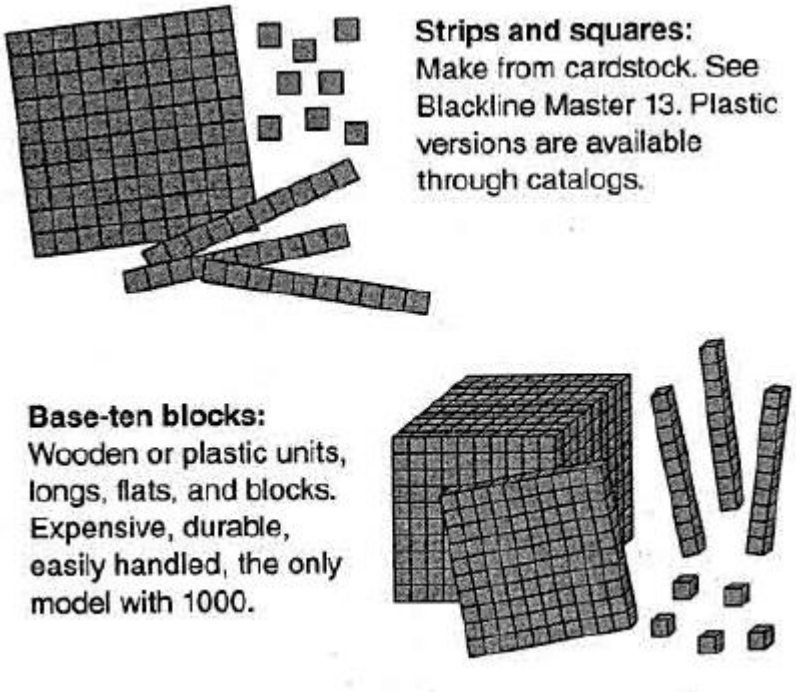


- Van de Walle, Lovin, Karp, & Bay-Williams., 2014

As children become more and more familiar with these models, collections of tens can be made in advance by the children and kept as ready-made tens. This is a good transition into the pre-grouped models.

Pre-grouped or Trading Models

Models that are pre-grouped are commonly shown in textbooks and are often used in instructional activities. Pre-grouped models cannot be taken apart. When 10 single pieces are accumulated they must be exchanged for a ten.



- Van de Walle, Lovin, Karp, & Bay-Williams., 2014

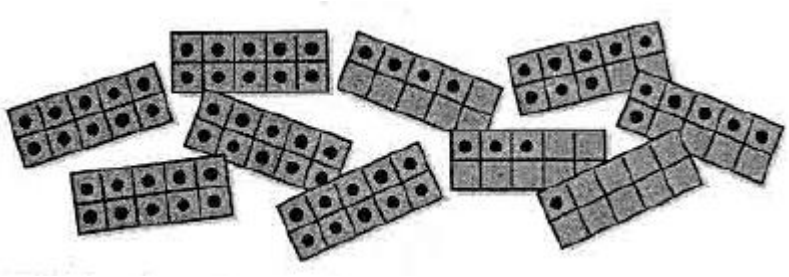
With pre-grouped models you need to make extra efforts to make sure that children understand that a ten piece really is the same as 10 ones. Here children combine multiplicative understanding (each piece is ten times the value of the place to the right) with a positional system (each place has a value) - something hard to do prior to multiplication!

Reflection

- What may be some disadvantages to using the pre-grouped physical models?

LITTLE TEN FRAMES

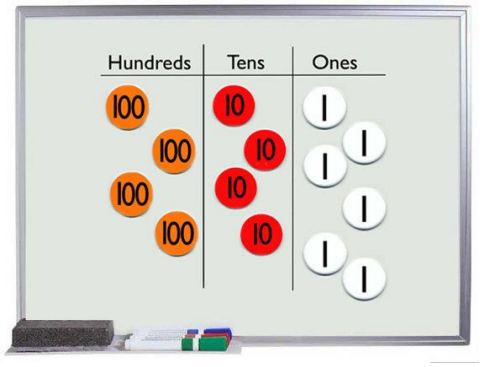
Little ten frames are less common but are very effective. If children have been using ten frames to think about numbers to 20, the value of the filled ten frame may be more meaningful than the ten rods and squares of base ten materials.



Although the ones are fixed on the cards, this mode has a distinct advantage of always showing the distance to the next decade. For example, when 47 is shown with four ten cards and a 7 card, a child can see that three more ones will make five full cards, or 50. This aids in using an open number line for addition.

Non-proportional Models

Non-proportional models can be used by children who understand that 10 units make a “ten”. In non-proportional models, the ten is not physically ten times larger than the one, such as in money, an abacus, or different chips assigned to different place values. Non-proportional representations should not be used to introduce place value concepts: they can be used once children have a conceptual understanding of the numeration system and need additional reinforcement.



Video: Brain Crossing

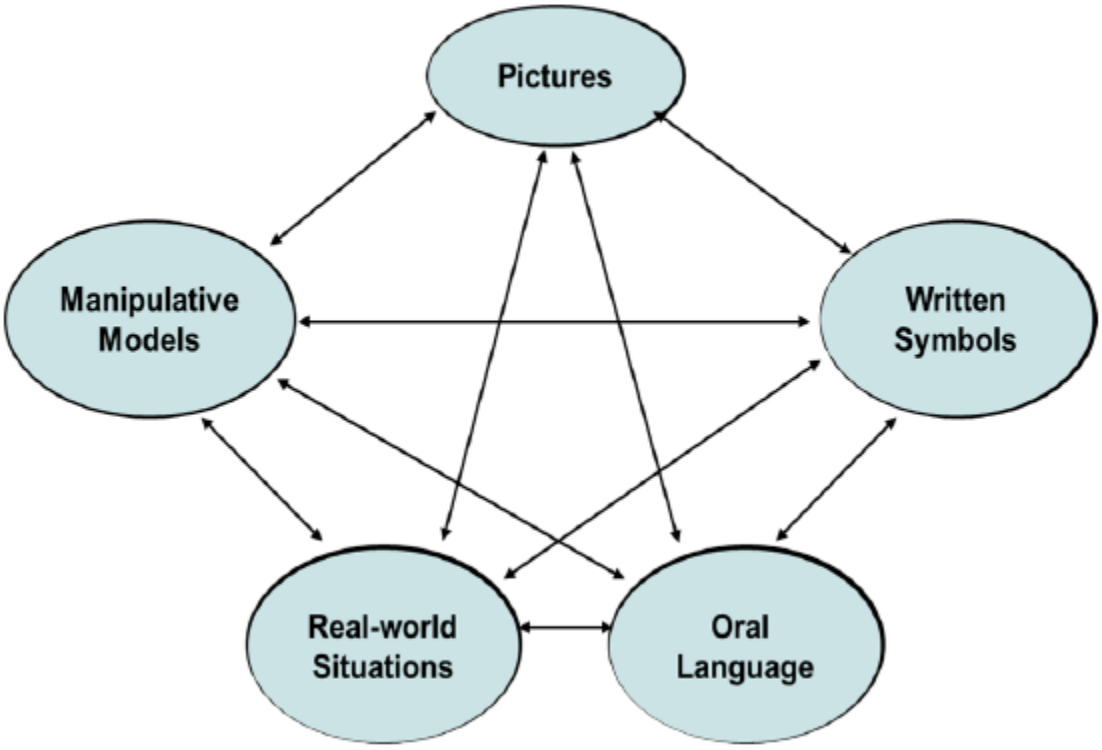
https://www.youtube.com/watch?v=qZBjub36Bvs

How does using both sides of our brain help us learn math better?

Discussion

The graphic below was introduced in previous math trainings. How does it relate to the brain crossing video?

Linking to Research/Literature
Connections between Representations



Adapted from Lesh, Post, & Behr, 1987

Developing Base-Ten Concepts

Connecting the important ideas of base ten concepts or grouping by tens with the oral and written names for numbers should be developed simultaneously with conceptual ideas.

Reflection

Think for a moment on how strange it must sound to say “seven ones.” Children have never said they were “seven ones” years old. The use of the word ten as a singular group name is even more mysterious. Consider the phrase “ten ones makes one ten.” The first ten carries the usual meaning of 10 things, the amount that is 1 more than 9 things. But the other ten is a singular noun, a thing. How can something the child has known for years as the name for a lot of things suddenly become one thing?

Because children come to their development of base-ten concepts with a count-by-ones idea of number, you must begin there. You cannot just impose grouping by ten on children. Children need to experiment with showing amounts in groups of like size and eventually come to an agreement that ten is a very useful size to use.

- Battista, M. *Cognition- Based Assessment & Teaching of Place Value: Building on Students' Reasoning*, 2012

Student Activity: Counting in Groups (1.NBT.2)

The activity is designed as an example of a first effort at developing grouping concepts.

Materials: countable items (tub of cubes, # of shoes in the class, # of crayons in crayon box)

1. Find a collection of items that children might be interested in counting. The quantity should be countable, somewhere between 25 and 100.
2. Pose the question, "How could we count our ____ in some way that would be easier than counting by ones?"
3. Whatever suggestions you get, try to implement them.
4. After trying several methods, have a discussion about what worked well and what did not.

If no one suggests counting by tens, you might casually suggest that as an idea to try.

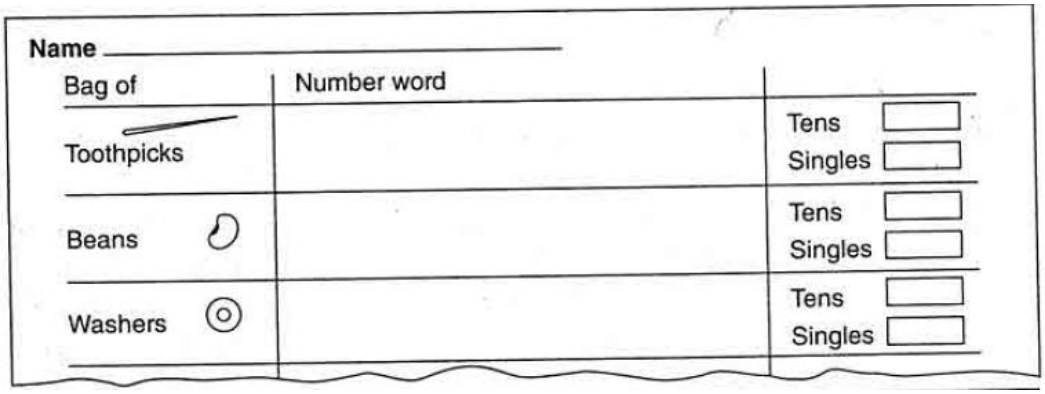
Student Activity: Groups of Ten (1.NBT.2)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Materials: bags of materials, recording sheet

In this activity children make groupings of ten and record or say the amounts. Number words are used so that children will not mechanically match tens and ones with individual digits. It is important that children confront the actual quantity in a manner that is meaningful to them.

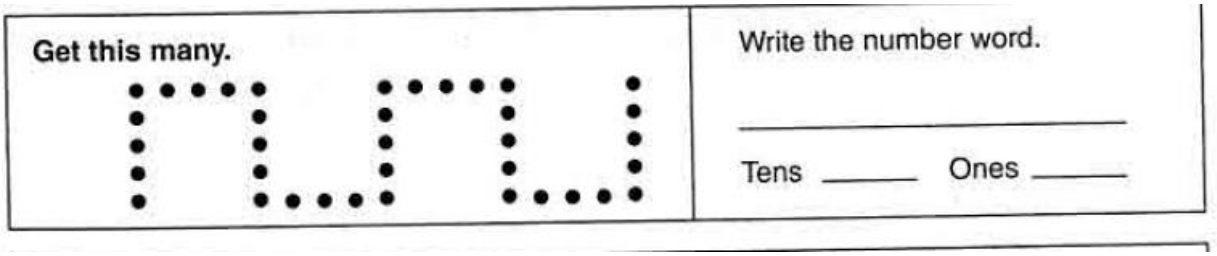
1. Prepare bags of different types of objects, such as toothpicks, buttons, beans, plastic chips, etc. The bags can be placed at stations around the room or given to pairs of children.
2. Children should have a recording sheet similar to the image below.
3. Children empty the bags and count the contents.
4. The amount is recorded as a number word.
5. Then the objects are grouped in as many tens as possible.
6. The groupings are recorded on the form.
7. After returning the objects to the bags, bags are traded or children move to another station.



Variations for Groups of Ten

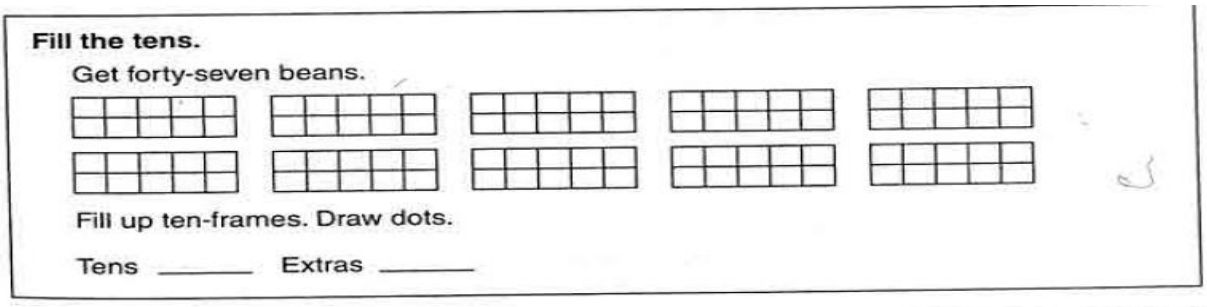
GET THIS MANY

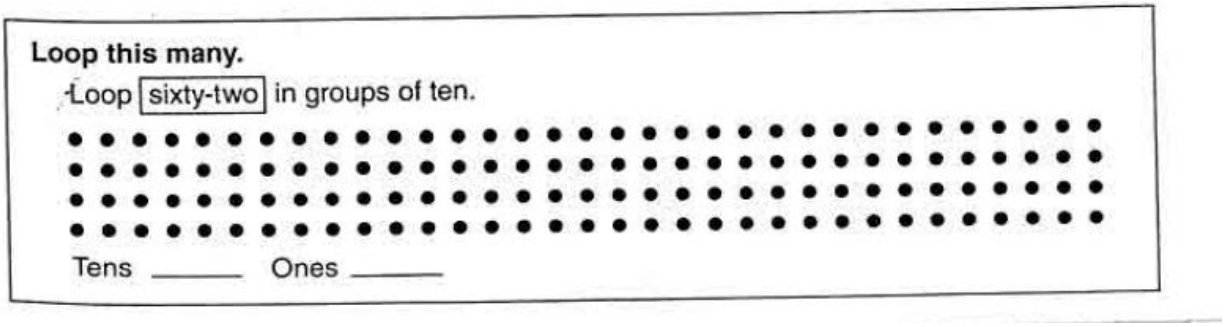
Children count the dots and then count out the corresponding number of objects. Provide small cups to put the groups of ten in. Notice that the activity requires children to first count the set in a way they understand (counting by ones), record the amount in words, and then make groupings.



FILL THE TENS OR LOOP THIS MANY

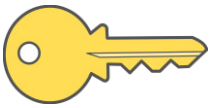
These sheets begin with a verbal name and children must count the indicated amount and then make groups.





Assessment Note

As you watch children doing these activities, you can learn a lot about their base-ten concept development. How do the children count the objects? Do they make groupings of ten? Do they count to 10 and then start again at 1? Children who do that are already using the base ten structure. But what you will more likely see early on is children counting a full set without stopping at tens and without any effort to group the materials in piles.



KEY IDEA #3

Children progress through three levels of understanding the concept of "ten" starting with understanding ten not as a unit but only as ten ones. They then move to seeing ten as a unit by relying on physical or mental reconstructions of models to help them work with units of ten. Finally, they are able to easily work with units of ten without the need of physical or mental reconstructions of base-ten models.

Small Group Discussion

You ask a child to count a small jar of beans. Then ask the child "If you were to place each group of 10 beans in a cup, how many cups would you need?" The child has no idea and makes a random guess.

- What does this tell you about the child's knowledge of place value?

Grouping Tens to Make 100

In second grade, numbers up to 1,000 become important. Here the issue is not one of connecting a count-by-ones approach to a group of 100, but rather seeing how a group of 100 can be understood as a group of 10 tens as well as 100 single ones, and seeing how a group of 1000 can be seen as a group of 10 hundreds as well as 1000 single ones.

Student Activity: Too Many Tens (2.NBT.1)

- Battista, M. *Cognition- Based Assessment & Teaching of Place Value: Building on Students' Reasoning*, 2012

Materials: 150 to 1000 of the same item (beans, cubes, etc), small cups, at least 10 large baggies.

In this activity it is important to use a groupable model so that children can see how the 10 groups of ten are the same as 100 individual items. This connection is often too implicit in the display of a hundreds flat or paper hundreds square in the pre-grouped base ten models.

1. Show students any quantity with 150 to 1000 items. (Ex. beans in a plastic bag)
2. Have the students record estimates of how many of the item are in the container. Discuss with children how they determined their estimates.
3. Give portions of the item to pairs or triads of children to put in cups of 10. Collect leftover beans and put these in groups of ten as well.
4. Ask," How can we use these groups of ten to tell how many _____ we have? Can we make new groups from the groups of ten? What is 10 groups of ten called?" Be prepared to have large baggies or containers in which the 10 cups can be placed. 5. When all groups are made, count the hundreds, the tens, and the ones separately. Record the totals on the board as "4 hundreds + 7 tens+ 8 ones".

Equivalent Representations

An important variation of the grouping activities is aimed at the equivalent representations of numbers. After children have just completed the "Groups of Ten" activity with a bag of objects pose the following task:

> "What is another way you can show 42 besides 4 groups of ten and 2 singles? Let's see how many you can find."
> "What is another way you can show 551 besides 5 groups of hundred, 5 tens, and 1 one? Let's see how many you can find."

Small Group Discussion

- What do you think would be most students' answer? Take a moment and work with a partner to see if you can come up with a list. Think about what knowledge or understanding is taking place as you are finding different representations.

Student Activity: Can you make the Link? (precursor activity for 2.NBT.7)

- Battista, M., 2012

Materials: items that can be partially grouped

1. Show a collection of materials that is only partially grouped in sets of ten. For example, you may have 5 bundled popsicle sticks of 10 and 17 individual sticks. Be sure the children understand that each bundle is a group of 10 sticks. (You can vary this activity by using larger numbers, i.e., 12 bundled popsicle sticks of ten and 17 individual sticks.)

2. Have children count the number of sticks and also count the singles, in any way they wish to count.

3. Ask, “How many in all?” Record all responses and discuss how they got their answers.

4. Change the groupings (make a ten from the singles or break apart one of the tens). Repeat the questions and discussion. Do not change the total number from one time to the next.

5. Once children begin to understand that the total does not change, ask in what other ways the items could be grouped if using tens and singles.

Student Activity: Three Other Ways (2.NBT.7)

Battista, M., 2012

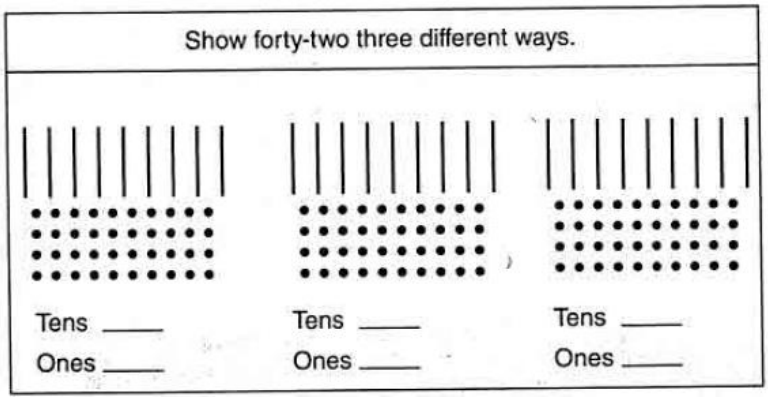
Materials: place value disks or base ten materials, student recording sheet.

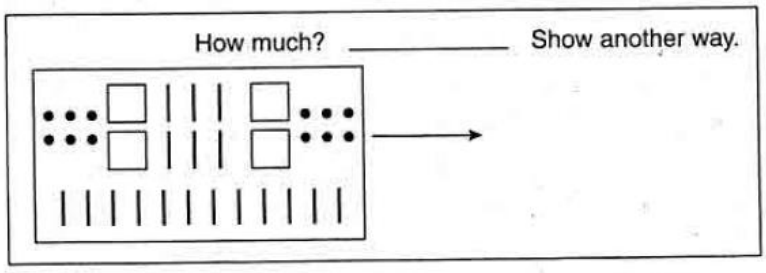
If you teach second grade, equivalent representations for hundreds as groups of tens can help children with the concept of a hundred as 10 tens.

1. Children work in groups or pairs. First they show a three-digit number on their desks with base ten materials in the standard representation.
2. They find and record at least three other ways of representing this quantity.

Variation - Challenge children to find a way to show an amount with a specific number of pieces. For example, ask children to show 463 with 31 pieces.

After children have had sufficient experiences with pre-grouped materials, a semi-abstract "dot-stick-square" notation can be used for recording ones, tens, and hundreds. Use the drawings as a means of telling the children which type of pieces to use to solve problems and also as a way for children to record results.





Participant/Student Activity: Base-Ten Riddles (2.NBT.7)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

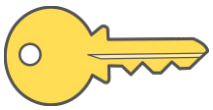
Base ten riddles can be presented orally or in written form. In either case, children should use base ten materials to help solve the riddles. After children solve the riddles, they can write new ones.

Take a few minutes and solve each of the following:

1. I have 23 ones and 4 tens. Who am I? _____
2. I have 4 hundreds, 12 tens and 6 ones. Who am I? _____
3. I have 30 ones and 3 hundreds. Who am I? _____
4. I am 45. I have 25 ones. How many tens do I have? _____
5. I have 13 tens, 2 hundreds, and 21 ones. Who am I? _____
6. If you put 3 more tens with me, I would be 115. Who am I? _____
7. I have 17 ones. I am between 40 and 50. Who am I? _____ How many tens do I have? ____

Reflection

Why would this activity be beneficial to students?



KEY IDEA #4

The groupings of ones, ten, and hundreds can be taken apart in different but equivalent ways.

Oral and Written Names for Numbers

It is vital that we give opportunities to help children connect oral and written names for numbers with their emerging base ten concepts of using groups of ten or one hundred as efficient methods of counting.

NOTE: The ways we say and write numbers are convention, not concepts. Children must learn these by being told rather than through problem-based activities.

Two-Digit Numbers

In kindergarten and first grade, children need to connect the base ten concepts with the oral number names they have used many times. They know the words but may have not thought of them in terms of tens and ones. In fact, early on they may want to write twenty-one as 201.

Almost always use base-ten models while teaching oral names. Initially rather than use standard number words, use the more explicit base-ten language (i.e. “4 tens and 7 ones” instead of “forty-seven”) Base-ten language is rarely misunderstood. When it seems appropriate, begin to pair base-ten language with standard language.

Emphasize the teens as exceptions. Acknowledge that they are formed “backward” and do not fit the patterns.

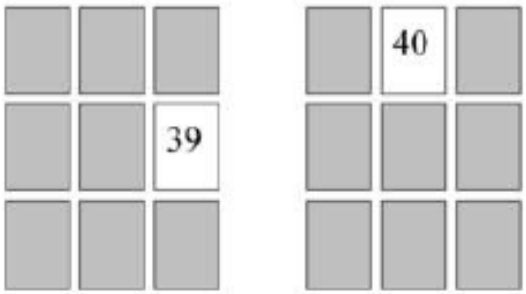
-Van De Walle, Lovin, Karp, & Bay Williams, 2014

TN Department of Education

Student Task: Crossing the Decade Concentration

You will need to create a set of number cards for each of the pair of numbers that cross the decade, i.e., 19 and 20, 29 and 30, 39 and 40, 49 and 50, etc.

Students place all the number cards that end with "_9" face down in an 3x3 array on the left and all the number cards that end with "_0" face down in a 3x3 array on the right. Working in pairs or trios, students take turns. The first student selects a card from the left array, stating the number name and the counting number that follows ("I have 39, I need 40").



He or she then picks one card from the array on the right (the "_0" numbers), hoping to find the target number. If the student does not find a pair, both cards are replaced face down in their original spots. It is now the second student's turn to choose a card from the "_9" array and to try to find the appropriate "_0" card. Students should try to remember where each number is located. (The game is called "Concentration" not "Guessing.")

When a student finds a matching pair he or she keeps that pair of cards. Play continues until all cards have been matched. The student with the most matched pairs wins.

TN Department of Education

IM Commentary

- One of the most common areas that young children struggle with when learning to count forward is crossing from one family to the next, for example getting to 29 in the counting sequence and not knowing what comes next or stating a random decade number. This game supports student development in this area. Students should have beginning knowledge of the counting sequence beyond the "teen" numbers before playing this game.
- This game can be introduced whole group on the board by making slightly larger cards and using a sentence pocket chart (or magnetic tape on a magnetic board) to arrange the cards in two arrays face down against the board. The teacher then plays against the rest of the class, modeling the process of picking from the left to begin, stating the number name and the number after and then picking from the right.
- It is very important to train the students to draw a card from the left and state what they need before they draw from the right. This will encourage them to think about and problem solve the next decade number. When the students get in the habit of picking up two cards simultaneously the game become much more about luck (although they do have to confirm that it is a pair, so do get some practice) and students are less likely to internalize the information and use it when counting.
- Students who become proficient with playing the game to support counting forward can gain experience in backward counting by picking from the right array first (the "_0" numbers) and then looking for the correct "_9" number. Changing the cards to 20-21, 30-31, 40-41, 50-51 etc with the "_0" numbers on the left and the "_1" numbers on the right is very supportive for another common error in backward counting which is to leave out the decade number when counting backward ie. "33, 32, 31, 29, 28".

Solution

Cards will be matched 19-20, 29-30, 39-40, 49-50, 59-60, 69-70, 79-80, 89-90, 99-100. This game can be modified when playing for the first time or for a struggling student by only making cards up to 50 and making 2 of each pair.

https://www.illustrativemathematics.org/content-standards/tasks/405

Student Activity: Counting Rows of 10 (1.NBT.2)

- Van de Walle, Lovin, Karp, & Bay-Williams., 2006

1. Using a 10 X 10 array of dots on a projector, cover up all but two rows.

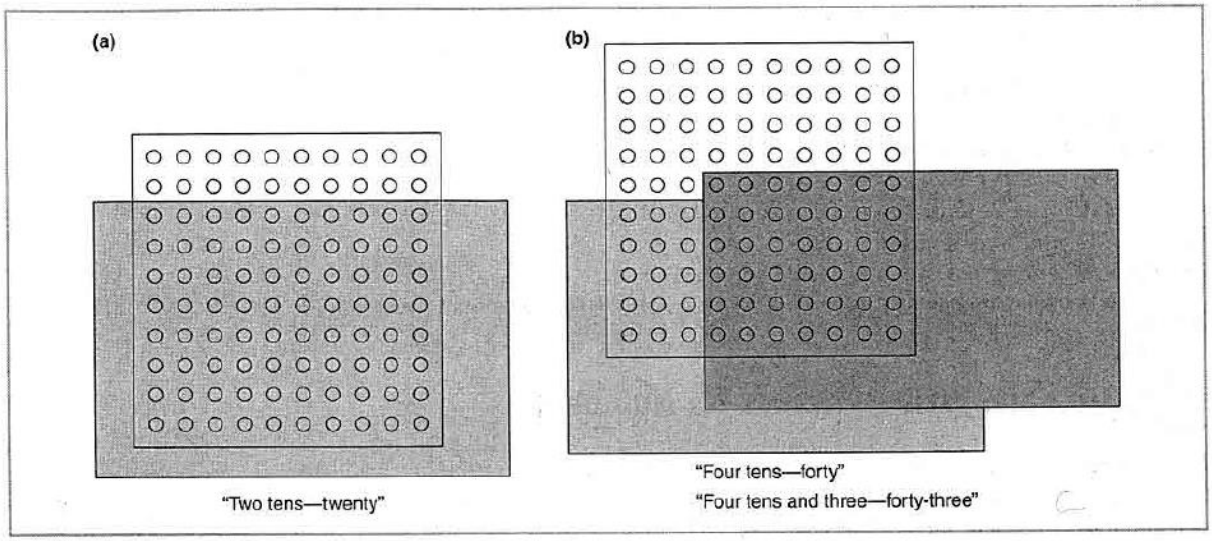
2. Ask, " How many tens? (2) Two tens is called twenty." Have the class repeat.

3. Show another row, and continue in this manner of questioning. Three tens is thirty. Four tens is forty. Five tens could have been fivety but is just fifty. The names sixty, seventy, eighty and ninety all fit the pattern.

4. Slide the cover up and down the array, asking how many tens and the name for that many. Some students may not hear a difference in numbers such as 50 and 15 or 60 and 16, so explicitly compare these words and clearly enunciate and even overemphasize the word endings.

5. Use the same 10 X 10 array to work on names for tens and ones. For example, show four full lines then expose one dot in the fifth row. Say "four tens and one. Forty-one."

6. Add more dots one at a time. "Four tens and two. Forty-two." When the pattern is established, repeat with other decades from 20 through 90.



Student Activity: Counting with Base Ten Models (1.NBT.5)

- Battista, M., 2012

Materials: Base ten materials (tens and ones) OR groupable tens and ones materials

1. Show some ten pieces on a project or carpet in a mixed arrangement.
2. Ask “How many tens?”
3. Add a ten or remove a ten and repeat the question.
4. Add some ones.
5. Always have the children give the base ten name and the standard name.
6. Continue to make changes in the materials displayed by adding or removing one or two tens and by adding or removing ones.

Avoid the standard left to right order for tens and ones so the emphasis is on the names of the materials not the order they are in.

Reverse the activity by having children use base ten blocks at their desks. You say a number - “Make 63”. The children make the number with their materials and then give the base ten name and standard name.

Two-Digit Numbers

Student Activity: Tens, Ones, and Fingers (1.NBT.2)

- Burns, Marilyn. *About Teaching Mathematics: A K-8 Resource*, 1998

Materials: Students

1. Ask the class, “Can you show 6 fingers (or any amount less than 10)?"
2. Ask, “How can you show 37 fingers?” Some children will figure out that at least 4 children are required.
3. Line up four children, have three of the children hold up 10 fingers while the fourth child holds up 7. Have the class count the fingers by tens and ones.
4. Ask other children to show different numbers.
5. Emphasize the number of sets of 10 fingers and the single fingers (base-ten language) and pair this with standard language.

NOTE: It is important occasionally to count an entire representation by ones. Remember that counting by ones is the young child’s principle linkage with the concept of quantity. For example, suppose you just had children use linking cubes to make 35. Try asking, “Do you think there are really 36 blocks there?” Many children may not be convinced, so the counting by ones is very significant.

Three Digit Number Names (2.NBT.1 & 2.NBT.3)

The approach to three-digit number names is essentially the same as for two digit names.

- Show mixed arrangements of base-ten materials
- Have children write the base-ten name and the standard name.
- Vary the arrangements for one example to the next by changing only one type of piece.
- Have children at their desks model numbers that you give to them orally using the standard names.

Student Activity: Make Six Numbers (2.NBT.3)

Make Six Numbers

Materials: numeral cards (1-9)

1. Turn over three cards.
2. Use all three cards to make **six** different numbers.

   Example: 5 1 2 512, 521, 152, 125, 215, 251
3. Record your numbers.
4. Order the numbers from least to greatest.
5. Show each number using base ten blocks, in word form, and in expanded form.

   Example: 521  five hundred twenty one 500 + 20 + 1

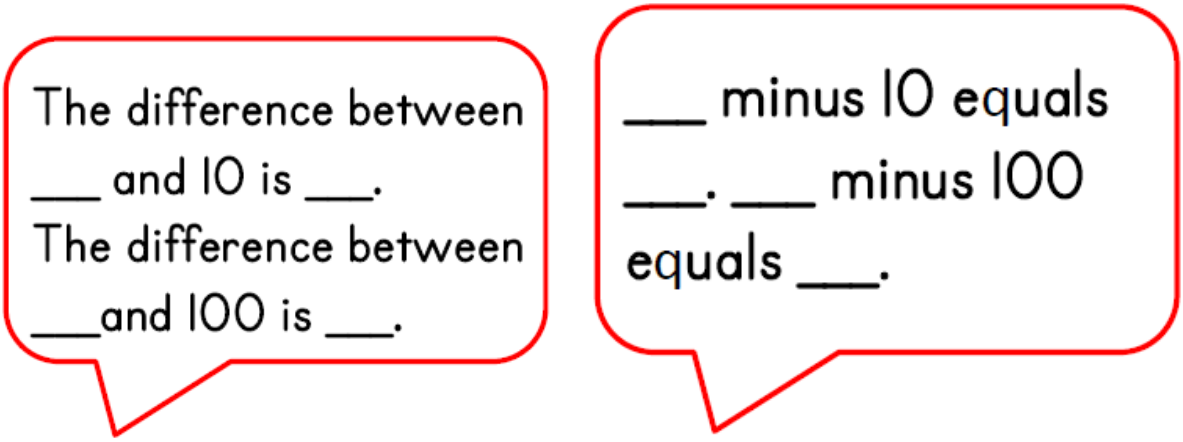
Retrieved from http://www.k-5mathteachingresources.com/support-files/make-6-numbers.pdf

Student Activity: Subtract 10 and 100 (2.NBT.8)

Subtract 10 and 100

Materials: set of numeral cards (0-9)

1. Work with a partner. Shuffle the cards and place them facedown. Turn over three cards to create a 3-digit number.
2. Subtract 10 from the number and record the equation. Subtract 100 from the original number and record the equation.
3. Change the order of the three cards to create a different 3-digit number. Subtract 10 and record. Subtract 100 and record.
4. Change the order of the three cards again to create a different 3-digit number. Subtract 10 and record. Subtract 100 and record.
5. Repeat steps 1-4 with a different set of three cards.



Retrieved from http://www.k-5mathteachingresources.com/support-files/subtract-10-and-100.pd

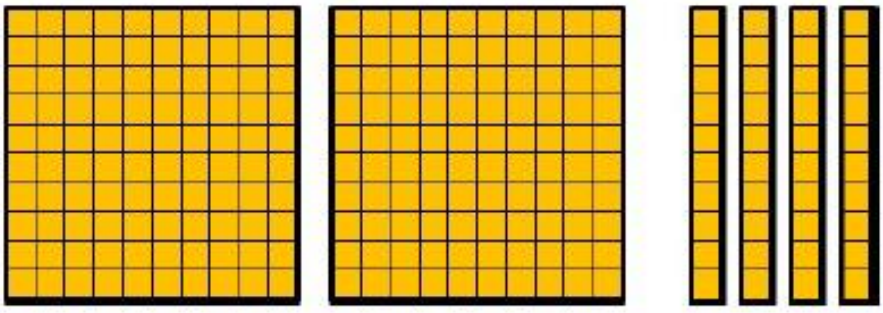
Student Task: Looking at Numbers Every Which Way Task

a. 127 is a number.
- Write it as a sum of 100's, 10's, and 1's.
- Write its name in words.
- Draw a picture to represent the number.
- Locate it on the number line.

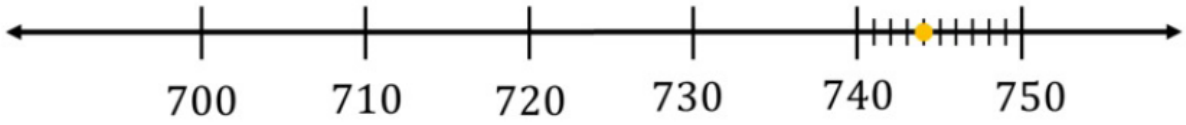
b. 500+60+8 is a number.
- Write it as a three-digit number.
- Write its name in words.
- Draw a picture to represent the number.
- Locate it on the number line.

c. Six hundred and nine is a number.
- Write it as a three-digit number.
- Write it as a sum of 100's, 10's, and 1's.
- Draw a picture to represent the number.
- Locate it on the number line.

d. The picture represents a number. The big square represents 100, the rectangle represents 10, and the small square represents 1.



e. A number is shown on the number line.



- Write it as a three-digit number.
- Write it as a sum of 100's, 10's, and 1's.
- Write its name in words.
- Draw a picture to represent the number.

IM Commentary

This task gives students the opportunity to work with multiple representations of base-ten numbers. The standard 2.NBT.3 asks students to read and write numbers to 1000 using base-ten numerals, number names, and expanded form. This task addresses all of these and extends it by asking students to represent the numbers with pictures and on the number line, which supports the understanding described in 2.NBT.1. Students who are still grappling with the meaning of base-ten numerals might benefit from having base-ten blocks on hand. Attached is a black line master for a place-value mat which can help scaffold students who are having trouble. Eventually, students should be able to do this task without concrete representations, however.

http://s3.amazonaws.com/illustrativemathematics/attachments/000/007/755/original/public_task_1236.pdf?1442499271

Discuss with your group:

What makes this a high-level task?

What can a teacher learn about his/her students' understanding of place value when looking at student responses?

Written Symbols

Place value mats are simple mats divided into two or three sections to hold ones and tens or one, tens, and hundreds pieces. You can suggest to children that the mats are a good way to organize their materials when working with base-ten blocks.

Assessment Note

Be aware of how easy it is for a child to show a number on a mat using base-ten blocks and learn to write the number without any understanding of what the number represents. First and second grade textbooks often show a picture of base ten materials and have children record numbers in this manner:

7 tens and 3 ones is 73 in all.

Reflection

- What strategies do you use to avoid this problem and to help show true understanding?

Here is one idea to the address the issue:

As children use their place-value mats, they can be shown how the left-to-right order of the pieces is also the way that numbers are written. To show how numbers are built, use the Place Value (or Hide Zero) cards (Module 2) and as children place the materials of a number (i.e., 457) on the mat they also place the matching card (400, 50, and 7) below the materials. Then, starting with the hundreds card, layer the others on top, right aligned.

This approach will show how the number is built while allowing children to see the individual components of the number. This is especially helpful when there are zero tens. The place value mat and the matching cards illustrate the important link between the base ten models and the written form of the numbers.

KEY IDEA #5

Children's ability to label the tens place and the ones place or to count by tens does not guarantee they understand that one ten is the same as ten ones.

Patterns and Relationships with Multi-digit Numbers

The Hundreds Chart

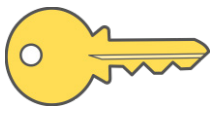
We want our children to look at patterns in our number system and how numbers are related. In particular, we are interested in helping children develop an understanding of the relationships of all numbers to special numbers, called "benchmark numbers". These ideas begin to provide a basis for computation.

In first and second grades, children can use the hundreds chart to develop base ten understanding, noticing that jumps up or down are jumps of ten, while jumps to the right and left are jumps of one.

Student Activity: Finding Neighbors on the Hundreds Chart (precursor to 1.NBT.4)

Conklin, M & Sheffield S. *It Makes Sense! : Using the Hundreds Chart to Build Number Sense*, 2010

1. Begin with a blank or nearly blank hundreds chart. Circle a particular missing number.
2. Children are to fill in the designated number and its “neighbors”. This can be done with a full class or with blank hundreds chart worksheets. It is important to use a “think aloud” to describe what key features of the numbers you think about as you determine the missing number and its neighbors. If there are children in your class who are ready, allow them to share their strategies instead of you.
3. After children become comfortable naming the neighbors of a number, ask what they notice about the neighboring numbers. What about those numbers on the diagonal?
4. By discussing these relationships on the chart, children begin to see how the sequence of numbers is related to the numerical relationships.



KEY IDEA #6

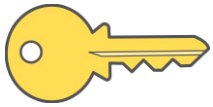
There are patterns to the way numbers are formed.

Student Activity: Models with the Hundreds Chart (1.NBT.4)

Conklin, M & Sheffield S., 2010

Many children at the first grade level will not understand the corresponding numeric relationships such as those discussed in the previous activity. In this activity number relationships on the chart are made more explicit by including the use of base ten materials.

1. Use any base-ten model for two-digit numbers with which the children are familiar. The little ten frames are recommended.
2. Give children one or more numbers to first make with the model and then find on the chart. Use groups of two or three numbers either in the same row or the same column.
3. Ask children, "How are the numbers in the row (or column) alike? How are they different?" To emphasize place value ask them "What place value is changing?"
4. Indicate a number on the chart. Ask children, "What would you have to change to make it into each of its neighbors?"



KEY IDEA #7

The positions of digits in numbers determine which value they represent.

Student Activity: The Thousands Chart (used to begin work with 2.NBT.2 and 2.NBT.3)

Van de Walle, Lovin, Karp, & Bay-Williams., 2006

It is also helpful for children to have a chart that extends to 200, even in the first grade. Perhaps a more powerful idea is to extend the hundreds chart to 1000.

1. Provide children with several sheets of the blank hundreds chart.
2. Assign groups of three or four children the task of creating a 1 to 1000 chart.
3. The chart is made by taping 10 blank hundreds charts together in a long strip.
4. Children should decide how they are going to divide up the task of filling in the chart with different children working on different parts of the chart.

The thousands chart should be discussed as a class to examine how numbers change as you count from one hundred to the next, what the patterns are, and so on. In fact, the earlier hundreds chart activities can be extended to a thousands chart.

Video: Lesson Idea—Trash Can Game

https://www.teachingchannel.org/videos/second-grade-math-lesson

Video Notes

Small Group Discussion:

- How does the repetition in the game allow for practice without redundancy?
- In what ways does the partner and whole group work help to scaffold learning?
- How are students developing oral language skills while playing either game?

Participant Activity: Play the Trash Can Game

Trash Can Game

_____ _____ _____ Trash Can _____ _____ (Try to make the number with the greatest value.)	_____ _____ _____ Trash Can _____ _____ (Try to make the number with the greatest value.)
_____ _____ _____ Trash Can _____ _____ (Try to make the number with the greatest value.)	_____ _____ _____ Trash Can _____ _____ (Try to make the number with the greatest value.)
_____ _____ _____ Trash Can _____ _____ (Try to make the number with the least value.)	_____ _____ _____ Trash Can _____ _____ (Try to make the number with the least Value.)
_____ _____ _____ Trash Can _____ _____ (Try to make the number with the least value.)	_____ _____ _____ Trash Can _____ _____ (Try to make the number with the least Value.)

101 and Out

Player 1_____

Roll #1	Roll #2	Roll #3	Roll #4	Roll #5	Roll #6	Total

Player 2_____

Roll #1	Roll #2	Roll #3	Roll #4	Roll #5	Roll #6	Total

Relationships with Benchmark Numbers

One of the most valuable features of both the hundreds chart and the little ten frame cards is how clearly they illustrate the distance to the next multiple of 10. Multiples of 10, 100, and occasionally other special numbers such as multiples of 25 are referred to as benchmark numbers. Children might learn to use this term as they work with informal methods of computation. No matter the terminology used, understanding how numbers relate to these special numbers is an important step in children's development of number sense and place value understanding.

In addition to the hundreds chart, the number line is an excellent way to explore these relationships.

Participant/Student Activity: Who am I? (2.MD.6)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

1. Sketch a line (or use a piece of cash register tape) and label 0 and 100 at opposite ends.
2. Mark a point with a "?" that corresponds to your secret number.
3. Children use estimation to try to identify your secret number.
4. For each estimate, place and label a mark on the line.
5. Continue marking each estimate until your secret number is discovered.
6. Have children explain how they are making their estimates. Highlight the use of any benchmark numbers in their estimations.
7. As a variation, the endpoints can be numbers other than 0 and 100. For example, try 0 and 1000, 200 and 300, or 500 and 800.

Participant/Student Activity: Who could they be? (2.MD.6)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

1. Label two points on a number line (not necessarily the ends) with benchmark numbers.
2. Show children different points labeled with letters. Ask what numbers these might be and why the children think that.
3. Question students about where other numbers might be (other than the letters), or how far apart do you think A and D are? Do you think A is more than 100?

Student Activity: Ordering Numbers Task

Malik is given a list of numbers:

$$1 \qquad 5 \qquad 10 \qquad 50 \qquad 100$$

He wants to include the following numbers so all numbers will be listed in order from least (on the left) to greatest (on the right):

$$49, 7, 22, 98, \text{ and } 3$$

Where in the list should he put each of these numbers?

IM Commentary

The purpose of this task is to give students an opportunity to compare numbers less than 100 to benchmark numbers. Even though a number line is not explicitly given in the task, it is useful for students to list the numbers in the order they would appear on the number line; this allows them to focus on the relative ordering without worrying about the exact placement on the number line. Students should also have a chance to do similar tasks with a number line.

1.NBT.3 asks students to record comparisons with the symbols $>, =$ and $<$. While this task does not ask for this, it helps build up an understanding of the relative magnitude of numbers which is a precursor to proficiency with the more abstract symbolic comparisons.

https://www.illustrativemathematics.org/content-standards/tasks/6

Discussion

How can you adapt this task for second grade?

Closing Activity

Connections to Real–world Ideas

As children study place value concepts, encourage them to see numbers in the world around them. Children in the first grade should be thinking about numbers to 100 and second graders should be thinking about numbers up to 1000.

Reflection

- Where are numbers like this?

Module Reflection

Think back to the areas we have explored concerning Place Value and Base Ten Concepts:

- Counting by Ones & Tens with Models and Words
- Base-Ten Grouping (Ones making Tens, Tens making Hundreds)
- Equivalent Representations
- Oral & Written Names for Numbers
- Patterns & Relationships with Multi-digit Numbers
- Relationships with Benchmark Numbers
- Connections to the Real World

Although we have looked at a variety of activities, this is not an exhaustive list.

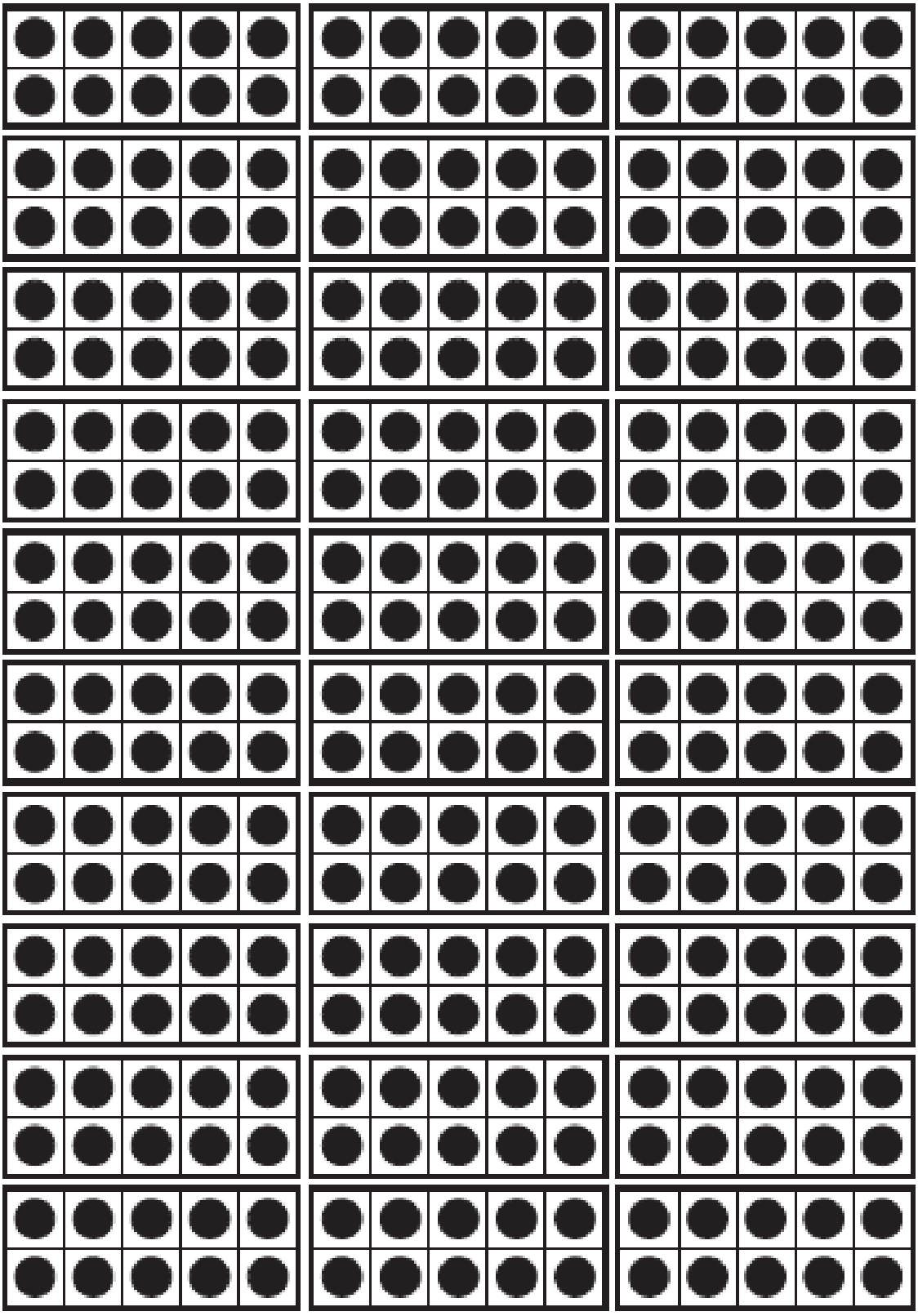
Reflection

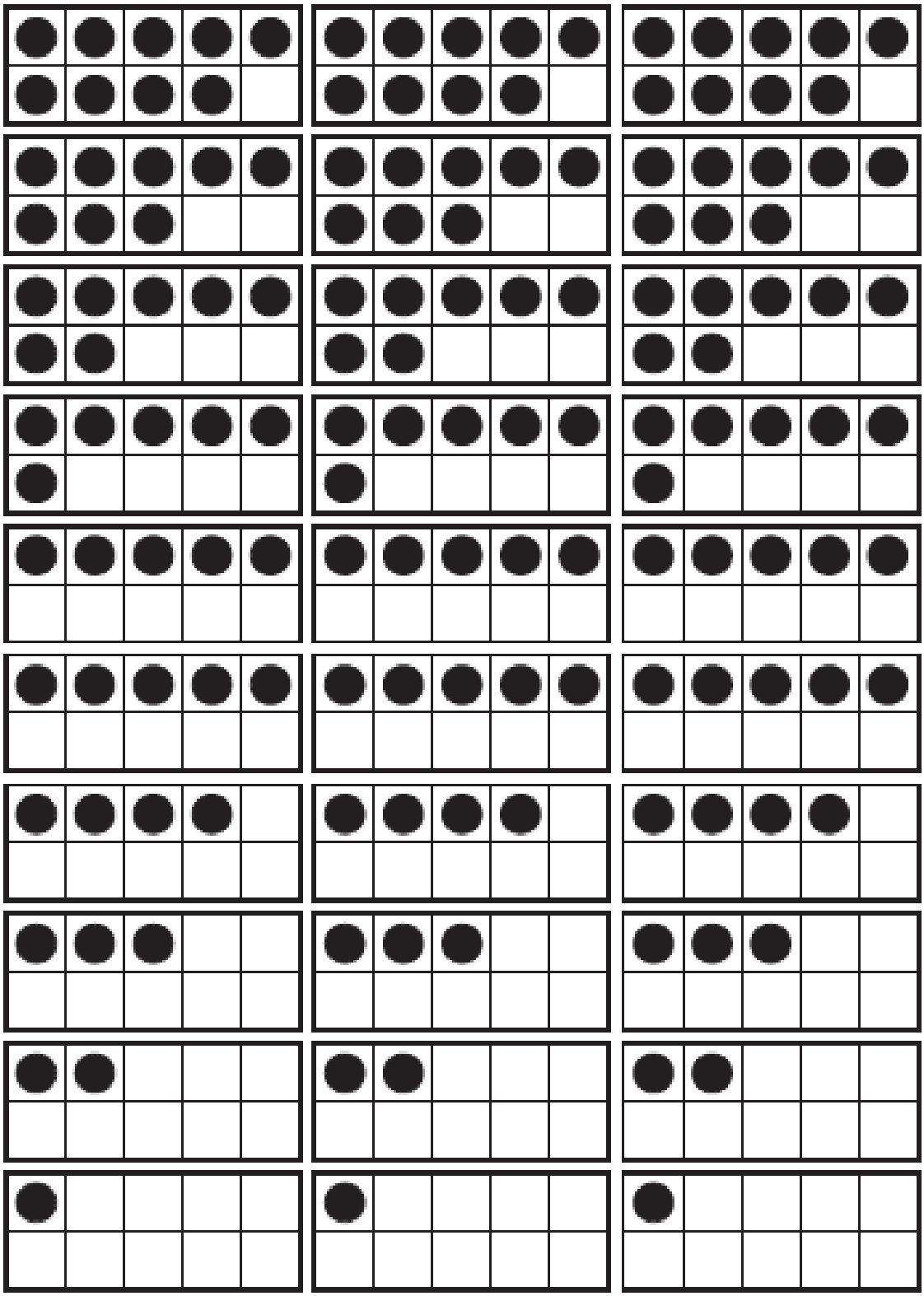
- What other activities do you use in your classroom to promote Place Value and Base Ten Concepts?

Discuss these strategies in your small group. Be prepared to share whole group. Record any ideas that you would like to incorporate in your classroom in the following pages.

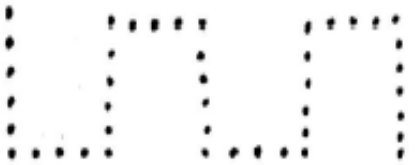
Collaborative Notes/Activities – Place Value & Base Ten

Module 3 Appendix



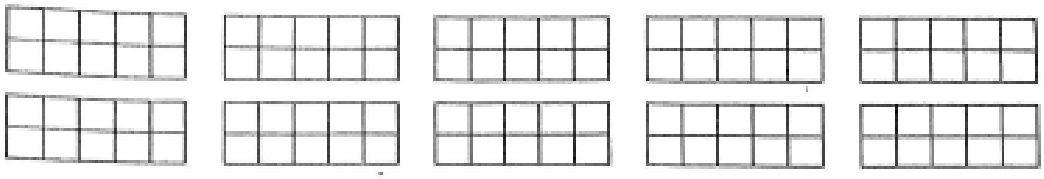


Name _____

Get this many. 	Write the number word Tens _____ Ones _____
Get this many. 	Write the number word Tens _____ Ones _____
Get this many.	Write the number word Tens _____ Ones _____
Get this many.	Write the number word Tens _____ Ones _____
Get this many.	Write the number word Tens _____ Ones _____
Get this many.	Write the number word Tens _____ Ones _____

Fill the tens.

Get _____ beans.

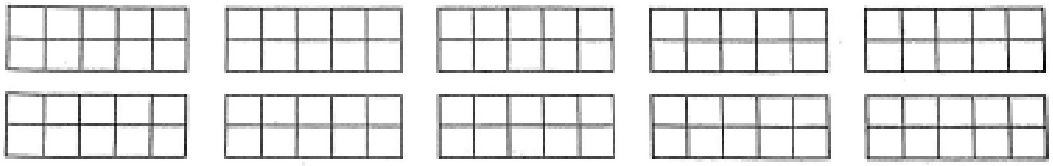


Fill up ten-frames. Draw dots.

Tens _____ Extras _____

Fill the tens.

Get _____ beans.

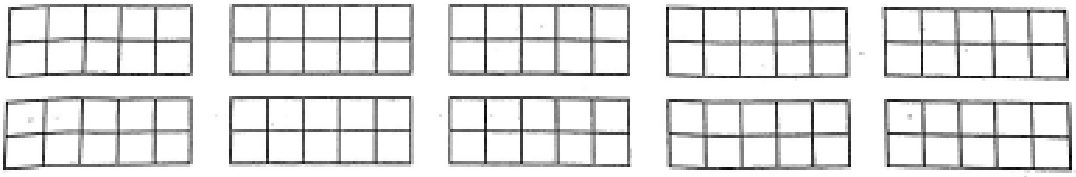


Fill up ten-frames. Draw dots.

Tens _____ Extras _____

Fill the tens.

Get _____ beans.

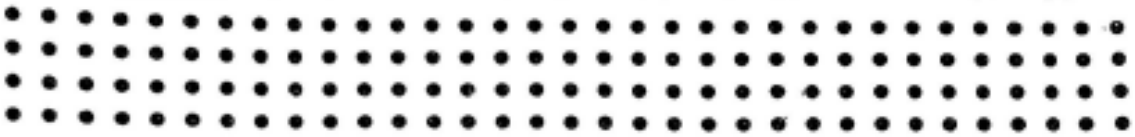


Fill up ten-frames. Draw dots.

Tens _____ Extras _____

Loop this many.

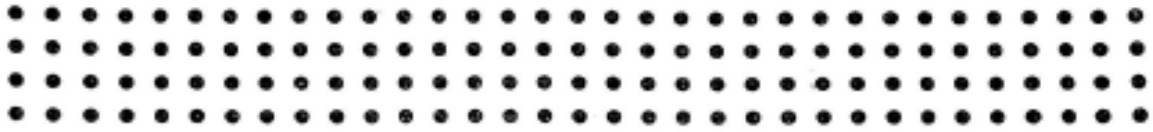
Loop [ ] in groups of ten.



Tens _____ Ones _____

Loop this many.

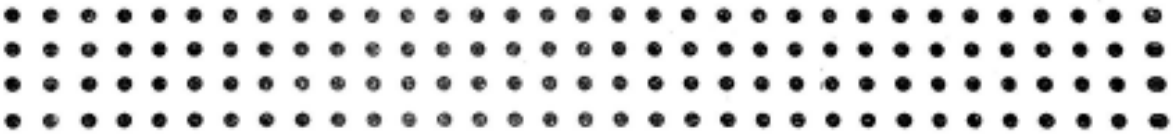
Loop [ ] in groups of ten.



Tens _____ Ones _____

Loop this many.

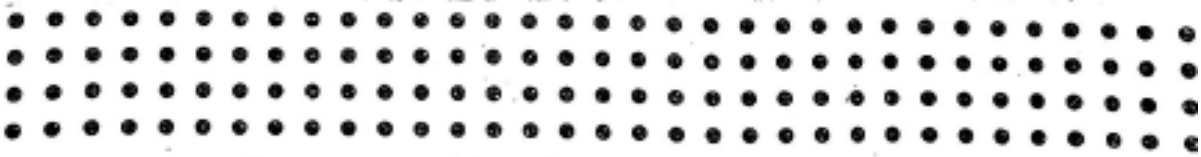
Loop [ ] in groups of ten.



Tens _____ Ones _____

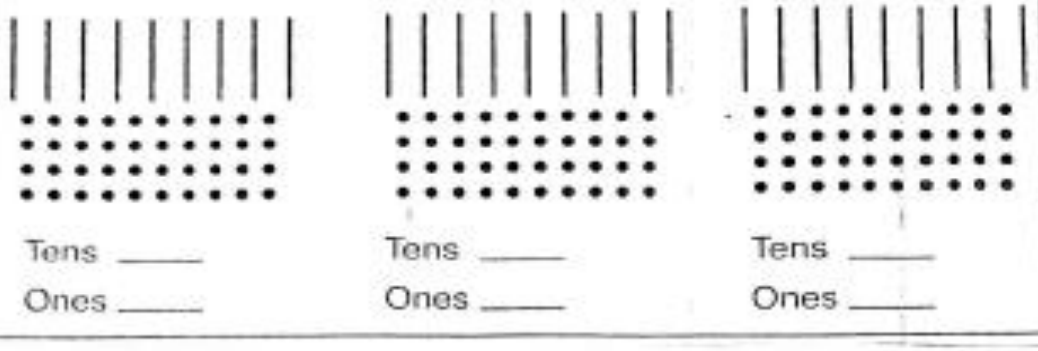
Loop this many.

Loop [ ] in groups of ten.



Tens _____ Ones _____

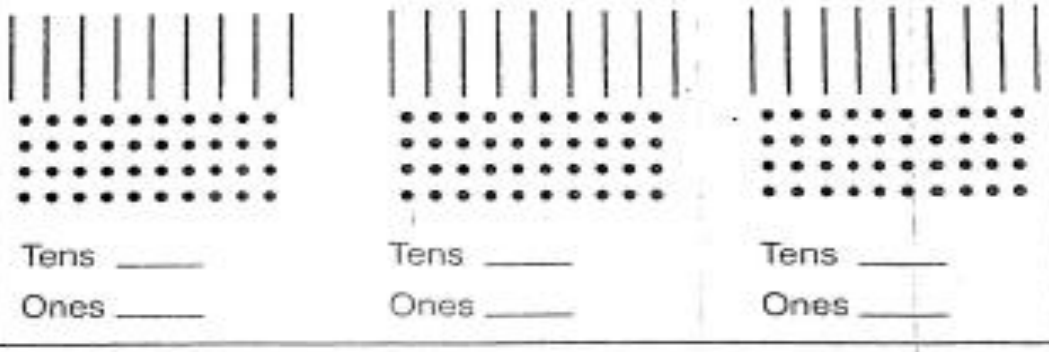
Show _____ three different ways.



Tens _____ Tens _____ Tens _____

Ones _____ Ones _____ Ones _____

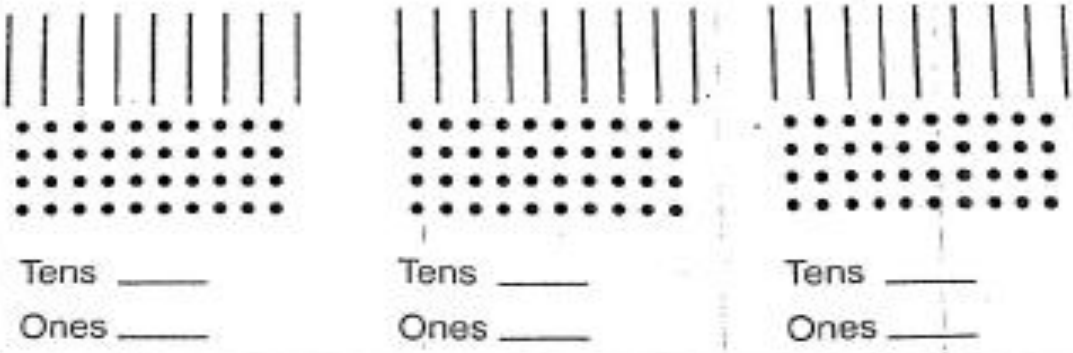
Show _____ three different ways.



Tens _____ Tens _____ Tens _____

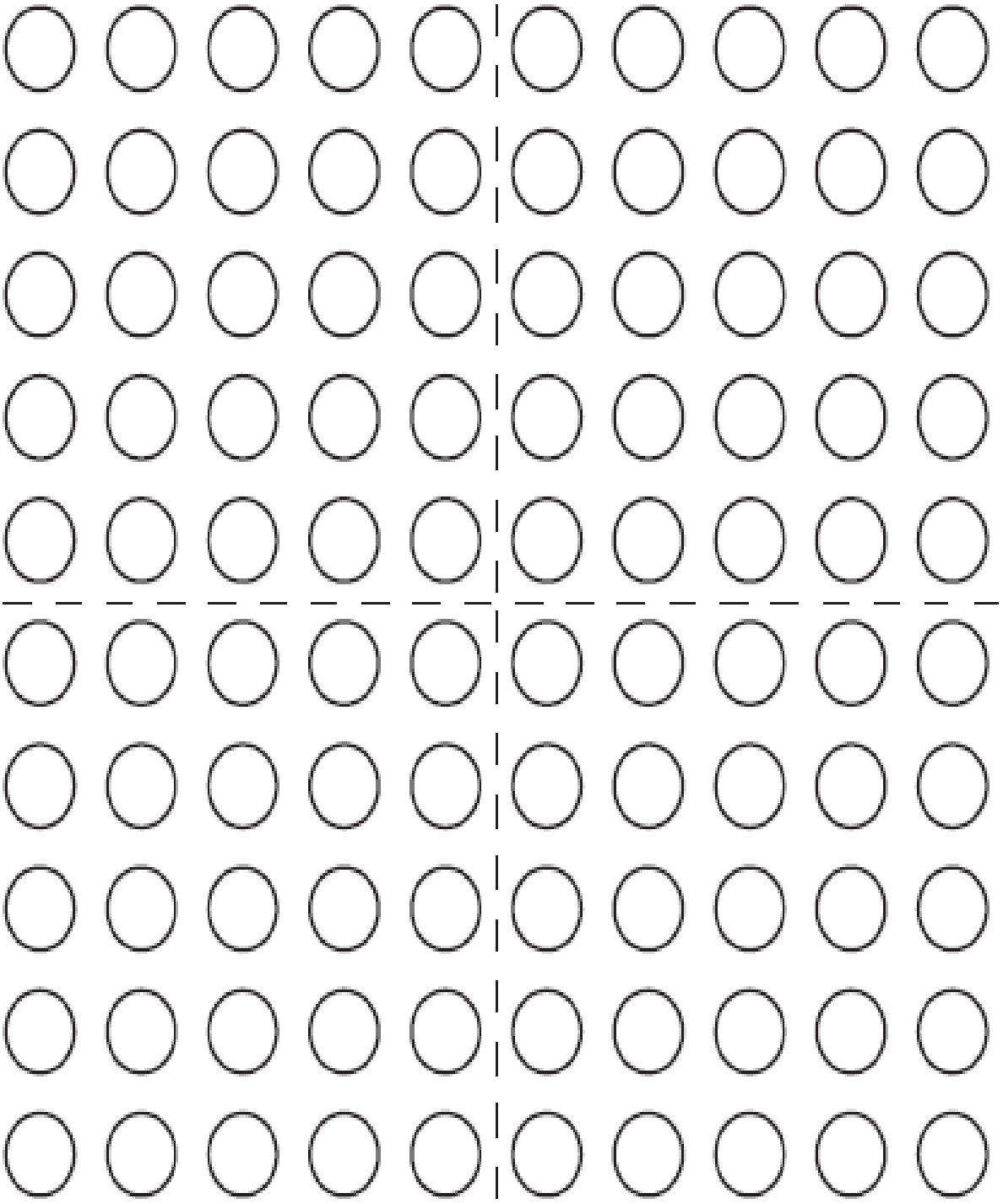
Ones _____ Ones _____ Ones _____

Show _____ three different ways.



Tens _____ Tens _____ Tens _____

Ones _____ Ones _____ Ones _____



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Trash Can Game

____ ____ ____ Trash Can ____ ____ (Try to make the number with the greatest value.)	____ ____ ____ Trash Can ____ ____ (Try to make the number with the greatest value.)
____ ____ ____ Trash Can ____ ____ (Try to make the number with the greatest value.)	____ ____ ____ Trash Can ____ ____ (Try to make the number with the greatest value.)
____ ____ ____ Trash Can ____ ____ (Try to make the number with the least value.)	____ ____ ____ Trash Can ____ ____ (Try to make the number with the least Value.)
____ ____ ____ Trash Can ____ ____ (Try to make the number with the least value.)	____ ____ ____ Trash Can ____ ____ (Try to make the number with the least Value.)

101 and Out

Player 1_____

Roll #1	Roll #2	Roll #3	Roll #4	Roll #5	Roll #6	Total

Player 2_____

Roll #1	Roll #2	Roll #3	Roll #4	Roll #5	Roll #6	Total

Module 4: Addition and Subtraction

Module 4: Addition and Subtraction

Objective

- To gain an understanding of flexible methods of addition and subtraction and the appropriate time to implement them
- To promote thinking about flexible methods and emphasize the need for a strong understanding of the operations coupled with place value

Tennessee Standards

1.NBT.4 Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.

1.OA.4 Understand subtraction as an unknown-addend problem. *For example, subtract 10-8 by finding the number that makes 10 when added to 8.*

1.OA.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

2.NBT.8 Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

2.NBT.9 Explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawings or objects.)

2.OA.1 Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

TEAM Alignment

- Standards and Objectives
- Presenting Instructional Content
- Activities and Materials
- Questioning
- Teacher Content Knowledge
- Thinking
- Problem Solving

Participant Activities:

- Participants will learn student activities to connect place value understandings to addition and subtraction
- Participants will read, reflect, and discuss three types of computation strategies
- Participants will consider ways to support invented computational strategies by students
- Participants will engage in activities that promote reasoning strategies for addition and subtraction of multi-digit numbers

Opening Activity

Let's Warm Up Our Minds!

https://www.youtube.com/watch?v=bsVBonvZuUg&app=desktop

Connecting Place Value to Addition and Subtraction

There is much more to learning about place value than having children state how many ones, tens, or hundreds are in a number. Children need ample time and opportunities to fully understand place value because it is such a complex concept - so complex that it took humans centuries to develop.

Researchers also suggest that problems involving addition and subtraction are a beneficial context for learning place value.

> "We know that children who only understand computation as a digit oriented exercise and not with full understanding of the numbers involved make many errors and have little judgment of the reasonableness of their answers. We must focus on helping children build flexible computational strategies using place-value knowledge. Here we lay the groundwork for developing both conceptual and procedural knowledge as we connect place value to addition and subtraction."
>
> Wright et al., 2008

Reflection

- Consider the scope and sequence you use. Does it align with the message of these quotes?

Student Activity: 50 AND SOME MORE (Supports 2.OA.1 and 2.NBT.7)

Retrieved from www.engageny.org

1. Say or write a number between 50 and 100.
2. Children respond with "50 and _____."
3. Any benchmark number can be used instead of 50. For example, you could use any number that ends in 50. Or as a challenge, use numbers such as 70 or 230.

Student Activity: THE OTHER PART OF 100 (Supports 2.OA.1 and 2.NBT.7)

Retrieved from www.engageny.org

Benchmark numbers are often used in computational strategies to make the computation easier to do. This activity is aimed at what may be the most important benchmark of all: 100.

1. Two children work together with a set of little ten-frame cards.
2. One child makes a two-digit number.
3. Then, both children work mentally to determine what goes with the ten-frame amount to make 100.
4. They write their solutions on paper and then check by making the other part with the cards to see if the total is 100.
5. Children take turns making the original number.

If children are adept at finding parts of 100, you can change the whole from 100 to another number. To start, try other multiples of 10, such as 70 or 80. Then, extend the whole to any number less than 100.

Participant Activity

Suppose that the whole is 83. Sketch four ten frame cards showing 36. Looking at your "cards", what goes with 36 to make 83?

Reflection

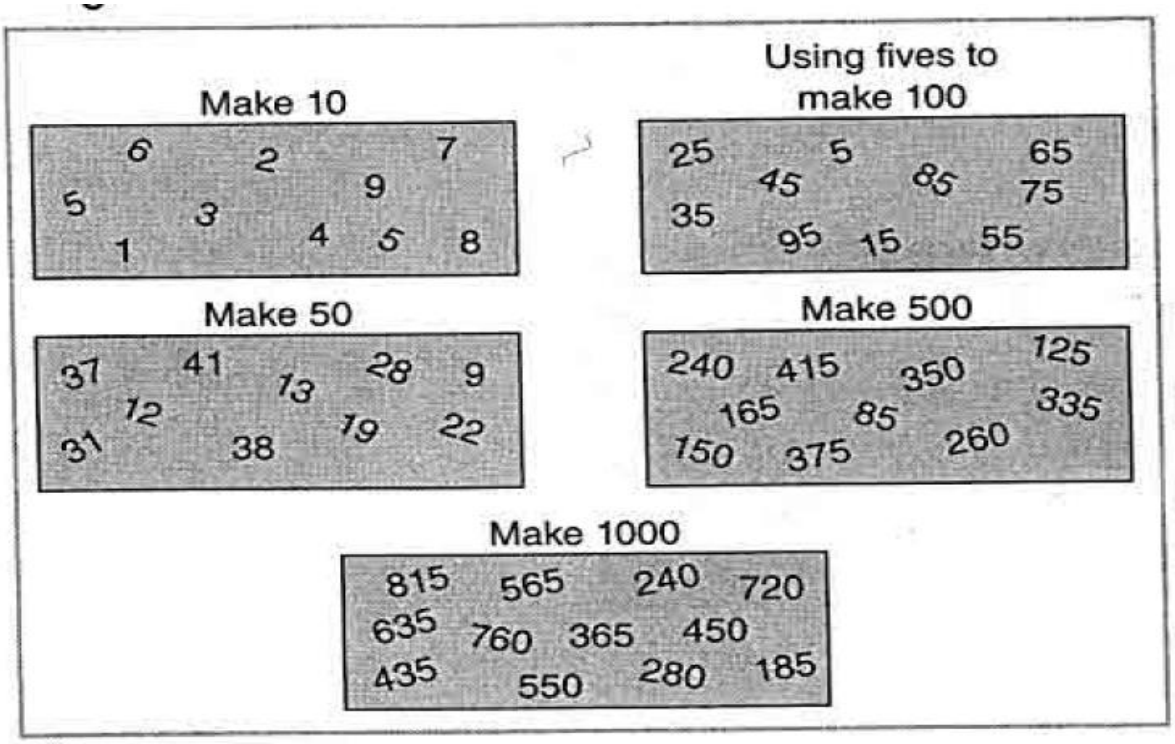
- What process did you go through to find your answer?

Student Activity: COMPATIBALE PAIRS (Supports 1.OA.4, 1.OA.6, 2.OA.1 and 2.NBT.7)

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Compatible numbers for addition and subtraction are numbers that go together easily to make benchmark numbers. Numbers that make tens or hundreds are the most common examples.

1. Searching for compatible pairs can be done as an activity with the full class.
2. The possible searches are at different difficulty levels.
3. Children name or connect the compatible numbers as they see them.



KEY IDEA #8

Flexible methods of addition and subtraction involve taking apart (decomposing) and combining (composing) numbers in a variety of ways.

Most methods for decomposing numbers are based on place value or compatible numbers.

Student Activity: CLOSE, FAR, AND IN BETWEEN

Van de Walle, Lovin, Karp, & Bay-Williams., 2014

This activity has children apply some of the same ideas about benchmark numbers that we have been exploring.

1. Put any three numbers on the board. Use two-digit numbers if those are more appropriate.
2. With the three numbers as reference, ask questions like the ones listed below. Encourage discussion of all responses.
    - Which two numbers are the closest? How do you know?
    - Which number is closer to ____?
    - Name a number between _____ and _____?
    - If these are "big numbers" what are some small numbers?
    - What are some numbers that are about the same?

Participant Activity

Working in pairs, your facilitator will give you three three-digit numbers. In partners, you will come up with a list of questions that could be used in this activity. Share and chart questions as a whole group.

Close, Far, and in Between
Numbers :
Questions

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Student Activity 11 More (1.NBT.5)

11 More

Number of players: 2 **Materials**: Counters of 2 different colors (one color per player), die

Instructions: Players take turns to roll a die and calculate the total of the number rolled plus 11.
After each roll the player places a counter on a corresponding number on the board. For example, if Player 1 rolls a 4 s/he may place a counter on any 15 on the board because 4+11=4+10+1=15. Play continues until one player has 4 counters in a row (horizontally, vertically, or diagonally).

12	14	13	15	16	17
14	12	16	15	13	12
17	12	14	13	15	16
16	15	13	14	12	17
17	16	12	15	14	17

I know that ll more than ___ is ___ because ___ plus l0 is ___ and l more is ___.

l0 more than ___ is ___. ll more than ___ is ___.

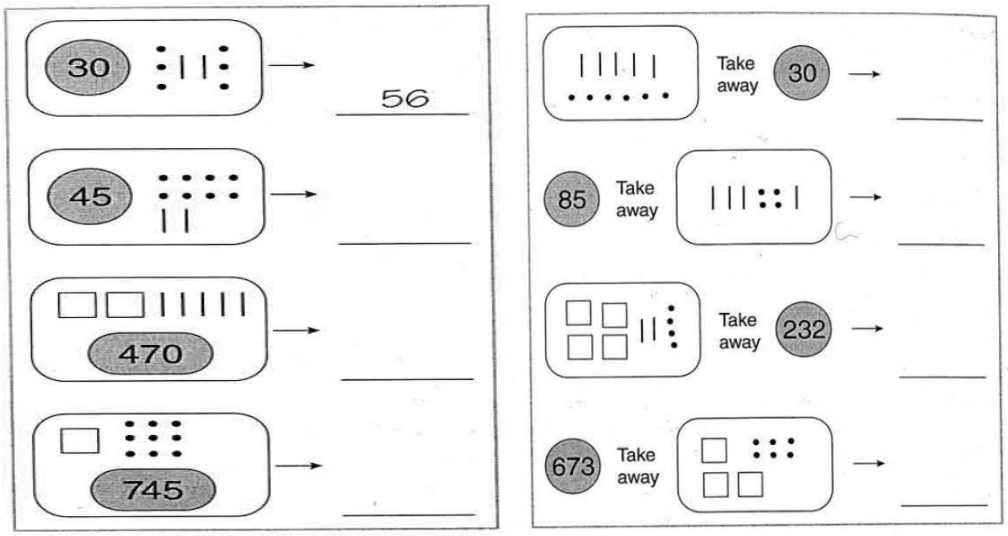
Retrieved from http://www.k-5mathteachingresources.com/support-files/11-more.pdf

Student Activity: NUMBER SQUARES, STICKS AND DOTS (2.NBT.7)

Battista, M. *Cognition- Based Assessment & Teaching of Addition and Subtraction: Building on Students' Reasoning*, 2012

This activity combines base-ten representations with symbolism.

1. On a circle of laminated construction paper, write a number with a dry erase marker.
2. Have students close their eyes as the teacher adds base ten materials. Add them in different orders and arrangements to promote flexible thinking.
3. Once students open their eyes, have them mentally compute the number.
4. Ask students to share their thought process.
5. If done in a small group, students may need to physically touch the pieces.
6. The same activity can be done for subtraction using a "take away" sign
7. You can do this as a whole class or a worksheet. If done as a worksheet, have students write about how they solved the problem. It is still important, however, to have a discussion with the class.



Student Activity: Base Ten Bag: Addition (2.NBT.7)

Base Ten Bag: Addition

Materials: Base 10 blocks (12 tens and 12 ones), paper bag, die

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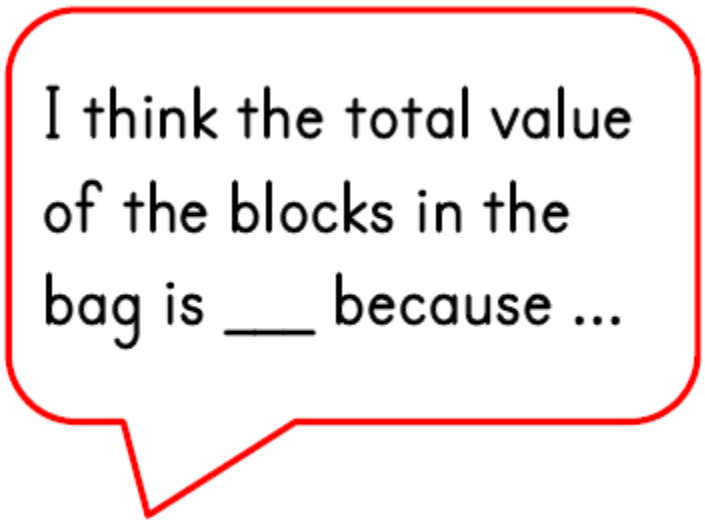
1. Work with a partner. Partner A: Roll a die and collect that number of tens. Roll again and collect that number of ones blocks.
2. Both partners: Record the total value of the blocks before placing them into the paper bag.
3. Repeat steps 1 and 2 with partner B rolling the die.
4. Figure out the total value of the blocks in the bag. Show your work.
5. Remove the blocks from the bag. Check your work by grouping and counting the tens and ones blocks.
6. Repeat steps 1-5 at least five times.



Retrieved from http://www.k-5mathteachingresources.com/support-files/base-10-bag-addition.pdf

Student Activity: HUNDREDS CHART ADDITION (1.NBT.4 and 2.NBT.2)

- Conklin, M & Sheffield S., 2010

1. Display a hundreds chart (or thousands chart) for all to see, or give each child their own individual chart.
2. Children use the hundreds chart to add two numbers.
3. There are many ways that children can use the hundreds chart to add two numbers, so class discussions are a must.
4. Have children work on one sum at a time and then have a discussion to compare the different methods.

The hundreds chart can be thought of as a stacked number line - one that emphasizes the distance from any number to the next multiple of 10. To begin, pose problems with relatively small second numbers such as:

17 + 14 23 +12 35 + 13 78 + 15

Many children will initially count on by ones from the first number, which is an indication that they may not understand how to count by tens from a given starting value (an important place-value concept).

Reflection

- How do the previous activities help promote mathematical understanding?

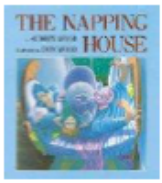
Moving To Computational Fluency

Addition and subtraction strategies that build on decomposing and composing numbers in flexible ways contribute to children's overall number sense. In most everyday instances, these alternative strategies for computing are easier and faster than the standard algorithms and can often be done mentally.

Literature Connection: **The Napping House** by Audrey & Don Wood.

A cumulative rhyme leads up to the consequences of piling too many sleepy people and animals in a cozy bed. The following task may be used after the story has been read.

The Napping House



Materials: copy of *The Napping House* by Audrey Wood

1. After listening to the story solve the following problem:

   When all the sleepers were piled up, how many feet were in the bed?

2. Use pictures, numbers and/or words to show how you solved the problem.

Retrieved from www.k5mathteachingresources.com

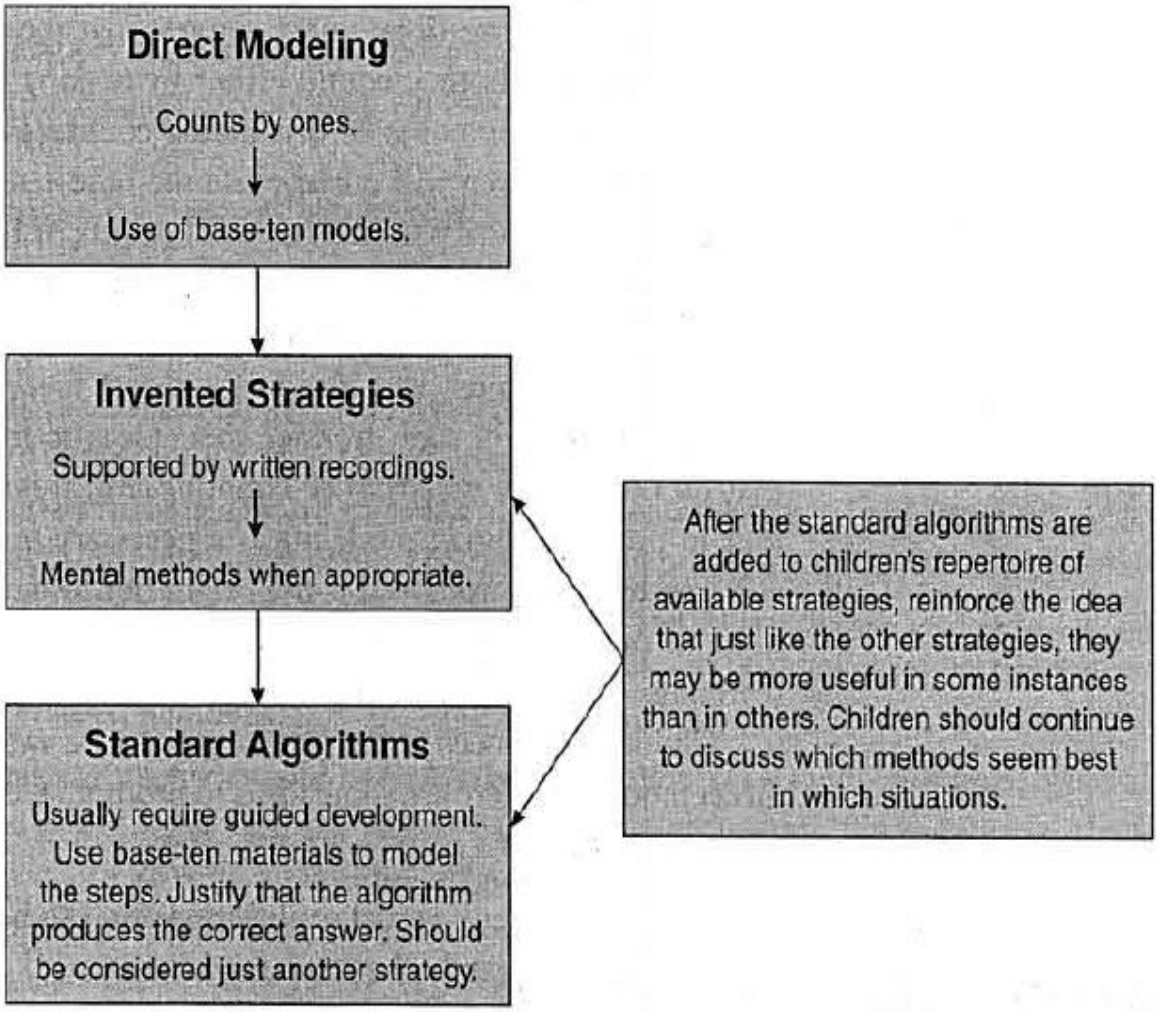
Participant Activity

Complete the following problem. Be prepared to discuss your reasoning with your small group.

Mary has 114 spaces in her photo album. So far she has 89 photos in the album. How many more photos can she put in before the album is full?

Three types of Computation Strategies

Van De Walle notes that there is a general instructional sequence for three types of computation strategies.



Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Direct Modeling

The developmental step that usually precedes invented strategies is called direct modeling: the use of manipulatives, drawings, or fingers along with counting to directly represent the meaning of an operation or story problem. This may be a child who has modeled numbers in a problem using counters then counted by ones to find an answer. Children who consistently count by ones most likely have not developed base-ten grouping concepts.

Children may need encouragement to move away from direct modeling. Here are some ideas to promote the transition from direct modeling:

- Record children's verbal explanation on the board in ways that they and others can model.
- Ask children who have just solved a problem with models to see if they can do it mentally.
- Ask children to make a written numeric record of what they did when they solved the problem with models. Explain that they are going to try to use the same written method on a new problem.

Note: Accepting direct modeling as a necessary developmental phase allows children who are not ready for more efficient methods a way to explore the same problems as classmates who have progressed beyond this stage.

Invented Strategies

We refer to any strategy, other than the standard algorithm, that does not involve the use of physical materials or counting by ones as an invented strategy. Tennessee State Standards describes these as "strategies based on place value, properties of operations, and/or the relationship between addition and subtraction". More specifically, children are expected to "develop, discuss, and use efficient, accurate and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations."

"There is mounting evidence that children both in and out of school can construct methods for adding and subtracting multi-digit numbers without explicit instruction. But not all children invent their own strategies. So strategies invented by class members are shared, explored, and tried out by others. However, children should not be permitted to use any strategy without understanding it."

Parrish, 2010

Contrasts between Invented Strategies and Standard Algorithms

There are significant differences between student-invented strategies and standard algorithms.

1. Invented strategies are number oriented rather than digit oriented. Using the standard algorithm for 45 + 32, children never think of 40 and 30 but rather 4 + 3.

2. Invented strategies represent a range of flexible options rather than "one right way". Invented strategies tend to change with the numbers involved in order to make the computation easier. The standard algorithm suggests using the same approach each time.

Participant Activity

Try each of these mentally. Be prepared to discuss with your group.

465 + 230 526 +98

Reflection

- Did you use the same method to solve each?

Participant Activity

The development and use of invented strategies support procedural proficiency. The positive benefits are difficult to ignore. With your table, generate a list of the benefits of using invented strategies. These will be charted as a whole group.

Invented Strategies

1. **Children make fewer errors**. Students tend to make fewer errors because they understand their own methods. Even when well-explained or modeled, many children do not understand the underlying concepts of standard algorithms. Not only do these children make more errors, but the errors are often systematic and difficult to remediate. Errors with invented strategies are less frequent and less systematic.
2. **Less reteaching is required**. Often teachers are concerned when children's early efforts with invented strategies are slow and time consuming. But the extended time for these early stages results in a meaningful and well-integrated network of ideas that is robust and long lasting. The increase in development time results in a significant decrease in reteaching and remediation.
3. **Children develop number sense**. Children's development and use of number–oriented, flexible algorithms helps them cultivate a rich understanding of the number system, especially place value concepts.
4. **Invented strategies are the basis for mental computation and estimation**. When invented strategies are the norm for computation, there is no need to talk about mental computation as if it were a separate skill. As children become more and more proficient with these flexible methods, they find they are able to use them mentally without having to write down even intermediate steps.
5. **Flexible methods are often faster than the standard algorithms.** Those who become adept with invented strategies will often perform addition and subtraction computations more quickly than those using a standard algorithm.
6. **Strategy inventions is itself an important process of "doing mathematics"**. Children who invent a computational strategy or who adopt a meaningful strategy developed by a classmate are involved in the process of sense making. This development of procedures is a process that is often hidden from children. By engaging in this aspect of mathematics, a significantly different and valuable view of "doing mathematics" is revealed to learners.
7. **Children who use invented strategies perform similarly or outperform their counterparts who are taught only standard algorithms.**

Reflection

- Do you feel the use of invented strategies is effective or ineffective? Why?

- How have you seen this reflected in your classroom and own teaching practices?

Standard Algorithm

Children who already know the standard algorithm may resist the development of more flexible strategies. What do you do then?

First and foremost, apply the same rule to standard algorithms as to all strategies. *If you use it, you must understand why it works and be able to explain it.* In an atmosphere that says, "Let's figure out why this works," children can profit from making sense of standard algorithms just as they can with invented strategies. But the responsibility for the explanations should be theirs, not yours!

As second grade teachers may feel the need to introduce the standard algorithm, it should be noted that students should spend a significant amount of time with invented strategies - months, not weeks. Teachers should begin with groupable base ten models, then go to the pre-grouped model, and last to the non-proportional model. Place value disks can be an excellent tool.

Video : Adding with Place Value Disks
https://www.youtube.com/watch?v=2aE6SmzX_7o

The following video shows how students begin making the connections of what they are doing while adding with the discs to a more abstract pictoral model. This is done after significant time has been spent with the place value disks.

Video: Pictoral Representations
https://www.youtube.com/watch?v=3_r2aFlNzLc

Reflection

How can using this manipulative help a students' mathematical understanding of adding when composing a ten is required?

Mental Computation

What may be a mental strategy for one child may require written steps for another child. Initially, children may not be ready to do computations mentally, as they may still be at the direct modeling stage or need to notate parts of the problem as they think through it. As children become more adept, they can and should be challenged to do appropriate computations mentally.

Today's mathematics curriculum and instruction must focus on preparing students to be mathematically proficient and compute accurately, efficiently and flexibly. If our goal is to create students who meet the above criteria, we must provide opportunities for them to grapple with number relationships, apply these relationships to computation strategies, and discuss and analyze their reasoning. Number Talks are an excellent way to help our students. Here is an example of a 2nd Grade Number Talk. As you watch the video, consider the following questions.

Video: Second Grade Number Talk

Reflection

- How does the teacher record each strategy to provide access for the class?
- Which strategies are easiest for you to understand?
- Which strategies are more challenging to follow?
- What mathematical concepts are being built upon during the number talk?
- How did the teacher bring these ideas to the forefront of her class?

Participant Activity

Solve the following problem using mental math. Be prepared to show your strategy to the whole group.

342 + 153 + 481

Invented Strategies for Addition and Subtraction

> Children should be able to use strategies for addition and subtraction that they understand and can use efficiently. Your goal might be that each of your children has at least one or two methods that are reasonably efficient, mathematically correct, and useful with lots of different numbers. Expect different children to settle on different strategies that play to their strengths.
>
> Children do not spontaneously invent wonderful computational methods while the teacher sits back and watches. In various experimental programs, children tended to develop or gravitate toward different strategies, suggesting that teachers and the programs do have an effect on the methods children develop.
>
> Vershcaffel et al., 2007

Creating a Supportive Environment

Children who are attempting to investigate new ideas in mathematics need a safe and nurturing classroom environment in which they can take risks, test conjectures, and try new approaches.

At your table, generate a list of the ways you create a supportive environment. Are there things you specifically avoid? Things you purposefully do? These will be shared and charted together.

Ideas for Creating a Supportive Environment

- Avoid immediately identifying the right answer when a child states it. Give other children a chance to consider whether they think it is correct.
- Expect and encourage student-to-student interactions, questions, and discussions.
- Promote curiosity and openness to new ideas and trying new things.
- Talk about both right and wrong ideas in non-evaluative and non-threating ways.
- Move less sophisticated thinking to more sophisticated thinking through encouragement, coaching, or strategic questioning (Assessing and Advancing Questions).
- Use familiar context and story problems to build a background and connect to children's experiences.

Models to Support Invented Strategies

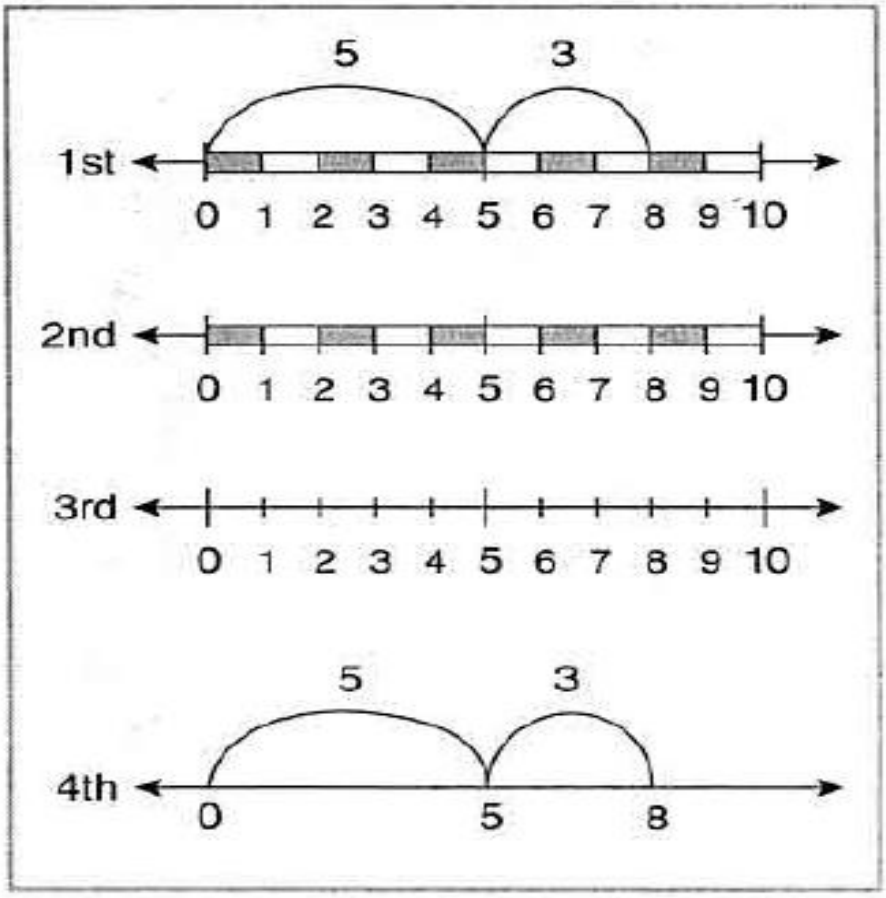
There are three common types of invented-strategy models that children come up with to solve addition and subtraction situations:

Strategy	Model
Split Strategy (Decomposition)	Arrows or lines to explicitly indicate how two computations are joined together.
Jump Strategy (Counting on or counting back)	Empty number line has been found to be more flexible and children are less prone to making computational errors when using it.
Shortcut Strategy (Compensation)	Because this strategy involves the flexible adjustment of numbers, you may find simply recording with an equation or an open number line to be most effective.

Number Lines

An innovative approach to computation is the use of open number lines to demonstrate addition and subtraction. A number line measures distances from zero the same way a ruler does, and can sometimes present conceptual difficulties to younger children, as they can mistakenly focus on the hash marks or numerals on a number line instead of the spaces (units of length) when counting.

Van de Walle provides a helpful visual of the sequence in which we should present number lines to make them meaningful for children.



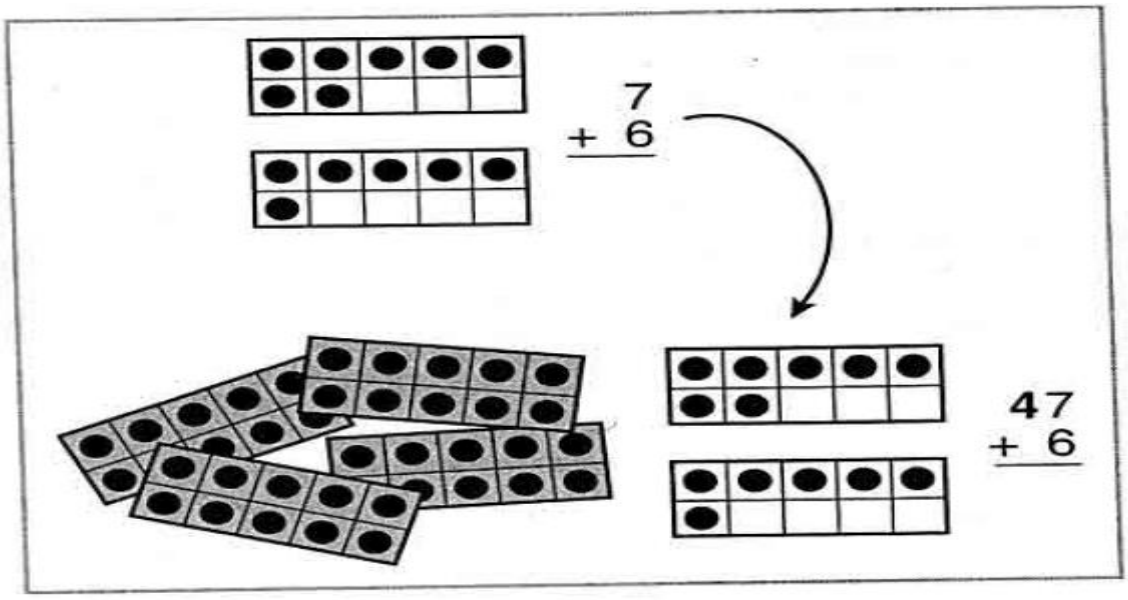
Reflection

- What are the subtle differences you notice in each of these number lines?
- Do you feel this is an effective way for children to communicate their reasoning? Why or why not?

Adding and Subtracting Single Digits

When adding or subtracting small amounts or finding the difference between two reasonably close numbers, many children will use counting to solve the problem. One goal should be to extend children's knowledge of basic facts and the ten structure of the number system so that counting is not required.

Students can use the Up Over 10 strategy when adding numbers that cross from one decade to another. For example, the computation 58+6 can we thought of as 58+2+4. Here, students add on to get 10, recognizing that 58+2=60, and then add the remaining singles, 60+4=64. Similarly, for subtraction, students can use the Down Over 10 strategy. For example, 53-7: take away 3 to get 50, then subtract four more to get 46.



Van de Walle, Lovin, Karp, & Bay-Williams., 2014

Adding and Subtracting Tens and Hundreds

As you move children from single-digit to two-digit numbers, adding and subtracting tens and hundreds is an important transition. Sums and differences involving multiples of 10 or 100 are easily computed mentally. Write a problem such as the following on the board:

300 + 500 + 20

Challenge children to solve it mentally. Ask them to share how they did it. Listen for the use of place value words: “3 hundred and 5 hundred is 8 hundred and 20 is 820.” Early problems should not require regrouping. Then, move to more difficult problems that do require regrouping. Using base ten models can help children think in terms of units of tens and hundreds.

Adding Two-Digit Numbers

Problems involving the sum of two two-digit numbers will usually produce a wide variety of strategies. Some of these strategies will involve starting with one or the other number and working from that point, either by adding on to get to the next ten or by adding tens to get from one number to the other.

Consider the following problem:

Two Scout troops went on a field trip. There were 46 Girl Scouts and 38 Boy Scouts. How many Scouts in all went on the trip?

Chart Activity: Let's consider four different strategies. Think about what it sounds like and how you would model it.

Strategy	What does it sound like?	Model
Add Tens, Add Ones, then Combine		
Add on Tens, then Add Ones		
Move Some to Make Tens		
Use a Nice Number and Compensate		

Participant Activity

Try using addition to solve this problem in as many different ways as you can. How many ways are like those we just discussed? Can you think of another way?

367 + 155

Your goal is to help children develop strategies that are efficient and that make sense to them. Listening to how some of their classmates use composition or decomposition can help others be more aware of different ways to adjust numbers.

Subtracting by Counting Up

This is a powerful way to subtract. Children working on the think-addition strategy for their basic facts can also be solving problems with larger numbers. The concept is the same.

Using a join with change unknown or missing part problem will encourage the counting up strategy.

Consider the following problem:

> Sam had 46 baseball cards. He went to a card show and got some more cards for his collection. Now he has 73 cards. How many cards did Sam buy at the card show?

Chart Activity: Let's consider three different strategies. Think about what it sounds like and how you would model it.

Strategy	What does it sound like?	Model
Add Tens, Add Ones, then Combine		
Add Tens to Overshoot, Then come back		
Add Ones to Make a Ten, then Tens and Ones		

Take Away Subtraction

Using take away subtraction is more difficult when done mentally. However, take away strategies are common, probably because many textbooks emphasize take away as the meaning of subtraction.

Consider the following problem:

> There were 73 children on the playground. 46 second grade students left the playground. How many children were still outside?

Chart Activity: Let's consider five different strategies. What does it sound like and how you would model it?

Strategy	What does it sound like?	Model
Take tens from the tens, then subtract the ones		
Take tens from the tens, then subtract the ones		
Take away tens, then ones		
Take extra tens, then add back		
Add to the whole if necessary		

KEY IDEA #9

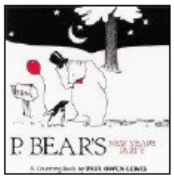
Flexible methods for computation require a strong understanding of the operations and properties of the operations, especially the commutative and associative property. Understanding how addition and subtraction are related as inverse operations is also important.

Literature Connection:

P. Bear's New Year's Party: A Counting Book by Paul Owen Lewis

A dapper polar bear has an elegant New Year's party and invites all of his animal friends—one whale, two horses, three cows, and so forth until midnight. As each party animal arrives, children learn to count. The following task can be used.

P. Bear's New Year's Party



Materials: copy of *P. Bear's New Year's Party* by Paul Owen Lewis

1. After listening to the story solve the following problem:

   How many guests came to P. Bear's party?

2. Use pictures, numbers or words to show your thinking.

Retrieved from www.k5mathteachingresources.com

Troubleshooting

Participant Activity

Some children may become confused when they hear their classmates talking about adding back numbers. For example, if they are solving 83-29, a child may say they took away 30 then added 1 back. Those students who are confused usually do not understand why they are adding back 1. They think that because you added 1 to 29 to make it 30 you should subtract 1 from the answer. Discuss with your group how you would help this student understand this strategy.

Student Activity: HOW FAR TO MY NUMBER?

-Battista, M., 2012

1. Children work in pairs for this activity.
2. Without communicating to each other, one child writes down a number less than 50 while the other child writes down a number greater than 50. You may choose to limit the size of the second number depending on the children.
3. The children work together to find out how much more must be added to the smaller number to get to the larger number.
4. Once an answer is determined, they should use little ten frame cards to represent the smaller number and the amount they found to see whether the total matches the larger number.

Another idea to help children think about using an adding–on approach is to show a number such as 28 with little ten frame cards and ask, “What goes with 28 to make 53?” You can do the same with three-digit numbers with or without the use of models.

Putting it all together with a Task Arc

A task arc is a set of related lessons which consist of several tasks and their associated lesson guides. The Lesson Progression Chart found in each task arc outlines the growing focus of content to be studied and the strategies and representations students may use. The lessons are sequenced in deliberate and intentional ways and are designed to be implemented in their entirety. It is possible for students to develop a deep understanding of concepts because a small number of standards are targeted. Lesson concepts remain the same as the lessons progress, however the context or representations change.

Note:

There are four task arcs for first grade. They can be found here: http://tncore.org/math/instructional_resources/grades/grade1.aspx.

There are three task arcs for second grade. They can be found here: http://tncore.org/math/instructional_resources/grades/grade2.aspx

Participant Activity

With a partner or small group, review the lesson progression chart in the task arc for your grade level. Record your “noticings” and “wonderings” below.

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Grade 1 Task Arc

The Relationship between Addition and Subtraction

http://www.edutoolbox.org/system/files/rasp_file/1stGradeTaskArc2.pdf

	Task 1 Apples and Oranges *Developing Understanding*	Task 2 Colored Beads *Developing Understanding*	Task 3 Playground *Developing Understanding*	Task 4 Pet Store *Developing Understanding*
Content	• Solves "take from" situational problems, result (difference) unknown • Introduces subtraction as an unknown-addend problem • Utilizes part-part-whole relationship	• Solves "take from" situational problems, result (difference) unknown • Explores subtraction as an unknown-addend problem • Utilizes part-part-whole relationship	• Solves "take from" situational problems, result (difference) unknown, change unknown • Explores subtraction as an unknown-addend problem • Utilizes part-part-whole relationship	• Solves "take from" situational problems, result (difference) unknown, change unknown • Establishes subtraction as an unknown-addend problem • Utilizes part-part-whole relationship
Strategy	• Counting Back from Whole • Counting On from Part • Math Facts, doubles	• Counting Back from Whole • Counting On from Part • Math Facts, doubles	• Counting Back from Whole • Counting On from Part • Math Facts, doubles +1	• Counting Back from Whole • Counting On from Part • Subtraction as undoing of Addition
Representations	• Starts with context that requires students to make sense of a situation • Asks students to represent the situation on the part-part-whole mapping device and to write equations	• Starts with context that requires students to make sense of information in a table to solve the problem • Asks students to represent the situation on the part-part-whole mapping device and to write equations	• Starts with context that requires students to make sense of a situation • Asks students to represent the situation on the part-part-whole mapping device and write equations	• Starts with context that requires students to make sense of information in a table to solve the problem • Asks students to represent the situation on the part-part-whole mapping device and to write equations

	Task 5 Subtraction as Addition *Solidifying Understanding*	Task 6 Bowls of Marbles *Developing Understanding*	Task 7 Jackson's Blocks *Developing Understanding*	Task 8 Sets of Equations *Solidifying Understanding*
Content	• Solidifies subtraction as an unknown-addend problem • Utilizes part-part-whole relationship	• Solves "take apart" situational problems, addend unknown • Applies concept of subtraction as an unknown-addend problem • Utilizes part-part-whole relationship	• Interprets "take apart" context • Utilizes part-part-whole relationship	• Applies understanding of part-part-whole relationship
Strategy	• Previously listed strategies	• Previously listed strategies	• Previously listed strategies	• Previously listed strategies
Representations	• Starts with a subtraction equation and a related unknown-addend equation • Asks students to use a subtraction equation to write unknown-addend equations and solve equations	• Starts with context that requires students to make sense of information in a table to solve the problem • Asks students to represent situation on the part-part-whole mapping device and to write related addition and subtraction equations	• Starts with a visual representation and challenges students to think flexibly about the quantities represented as they impose the meaning of the part-part-whole relationship • Asks students to write related addition and subtraction equations	• Starts with numbers, related equations, and a visual model and challenges students to think flexibly about the quantities and the way the quantities are represented • Asks students to represent sets of numbers with a diagram or to reference part-part-whole relationship and write related addition and subtraction equations • Repeated reasoning expected

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Grade 2 Task Arc

Addition: Flexible Thinking and Problem-Solving Strategies

http://www.edutoolbox.org/system/files/rasp_file/2ndGradeTaskArc-Addition-Flexiblethinking.pdf

	Task 1 **Prizes for the Class** *Developing Understanding*	**Task 2** **Collections** *Developing Understanding*	**Task 3** **What's Changing?** *Solidifying Understanding*	**Task 4** **Sit-Ups** *Developing Understanding*
Content	Explores place value, adding 10 and groups of 10 and adding numbers according to their place value by solving "put together" situational problems, whole unknown.	Establishes understanding of place value, adding 10 and groups of 10 and adding numbers according to their place value by solving "put together" situational problems, whole unknown.	Solidifies understanding of place value, adding 10 and groups of 10 by solving addition equations, sum unknown. Recognizes that only the tens place changes when adding 10.	Solves "put together" situational problems, whole unknown. Explores conservation, moving amounts between addends without changing the sum by solving "put together" situational problems, whole unknown.
Strategy	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts.	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts.	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts.	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers (moves amounts from one addend to another and adds).
Representations	Starts with context. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.	Starts with context. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.	Starts with a series of equations and asks students to solve for the sum. Asks students to analyze sets of equations and to determine how the sums change. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent addition equations.	Starts with context presented in the form of a table. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.

	Task 5 **Flowers** *Developing Understanding*	**Task 6** **Sports Equipment** *Developing Understanding*	**Task 7** **Fieldtrip to the Zoo** *Developing Understanding*	**Task 8** **Flexible Thinking** *Solidifying Understanding*
Content	Solves "put together" situational problems, whole unknown. Explores conservation.	Solves "put together" situational problems, whole unknown. Establishes understanding of conservation. Explores compensation (rounding up and adjusting for increase).	Solves "put together" situational problems, whole unknown. Establishes understanding of compensation.	Solidifies understanding of compensation, conservation, and place value, by solving addition equations, sums unknown.
Strategy	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers. Make a Ten from Ones (uses the ones to make a ten and then adds).	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers. Make a Ten from Ones. Make Friendly Numbers and Adjust (increases an addend, adds and then adjusts the resulting sum).	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers. Make a Ten from Ones. Make Friendly Numbers and Adjust.	Count On by Ten from a two-digit number. Add by Place Value to combine like amounts. Juggle Amounts to Make Friendly Numbers. Make a Ten from Ones. Make Friendly Numbers and Adjust.
Representations	Starts with context presented in the form of a table. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.	Starts with context presented in the form of a table. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.	Starts with context presented in the form of a table. Asks students to complete the table by filling in the missing sums. Asks students to use the part-part-whole map to construct a model or to use a hundreds chart to represent a situation and write equations.	Asks students to analyze sets of expressions, determine what changed in the sets, and then apply the same reasoning to another set of expressions.

Closing Activity

Think back to the areas we have explored concerning Addition and Subtraction Concepts:

- Connecting Place Value to Addition & Subtraction
- Computation Strategies
    - Direct Modeling
    - Invented Strategies
    - Mental Computation

Although we have looked at a variety of activities, this is not an exhaustive list.

Reflection

- What other activities do you use in your classroom to promote Addition and Subtraction Concepts?

Discuss these strategies in your small group. Be prepared to share whole group. Record any ideas that you would like to incorporate in your classroom in the following pages.

Collaborative Notes/Activities – Addition & Subtraction

Collaborative Notes/Activities – Addition & Subtraction

Module 4 Appendix

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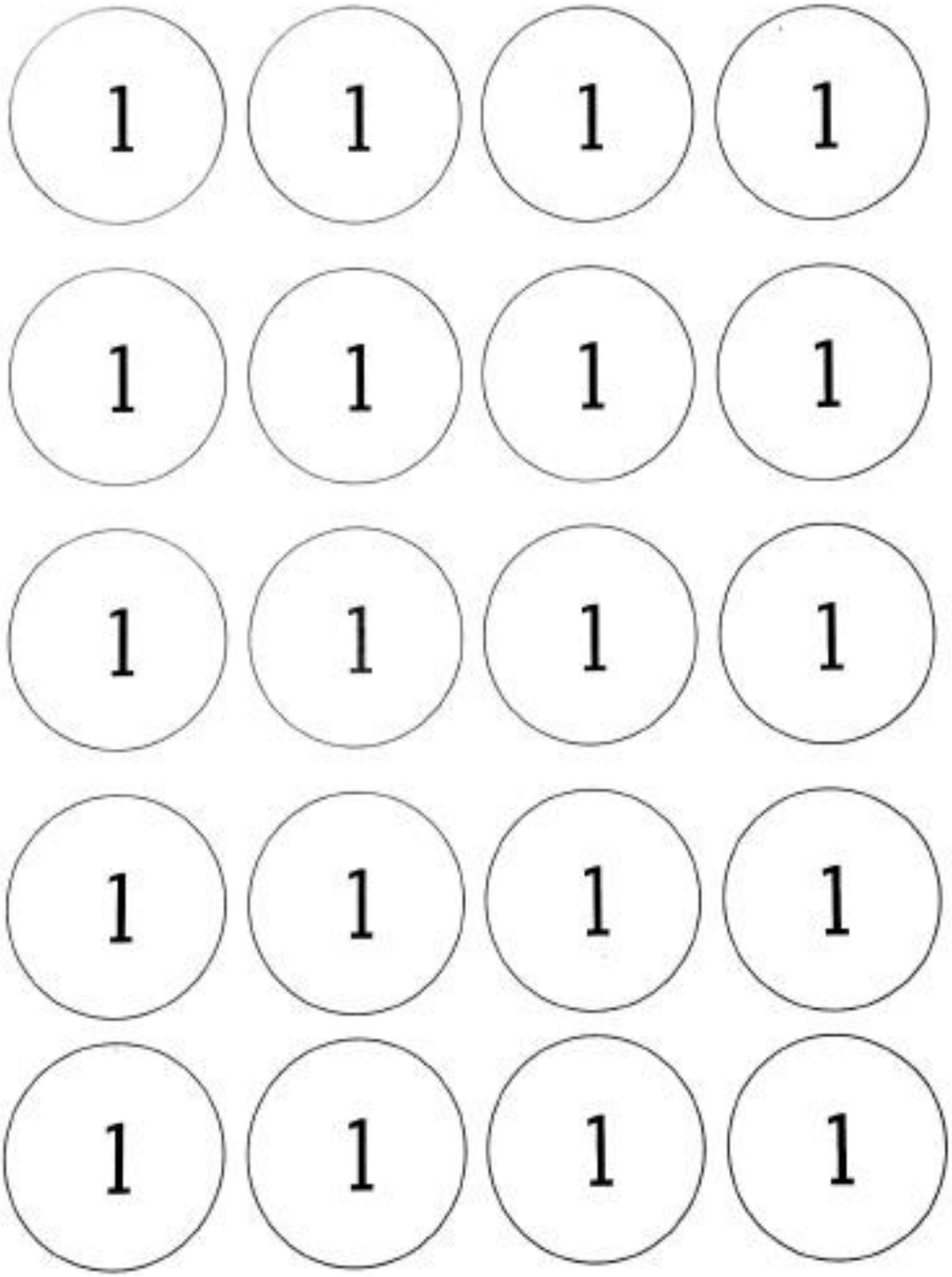
PLACE VALUE DISKS



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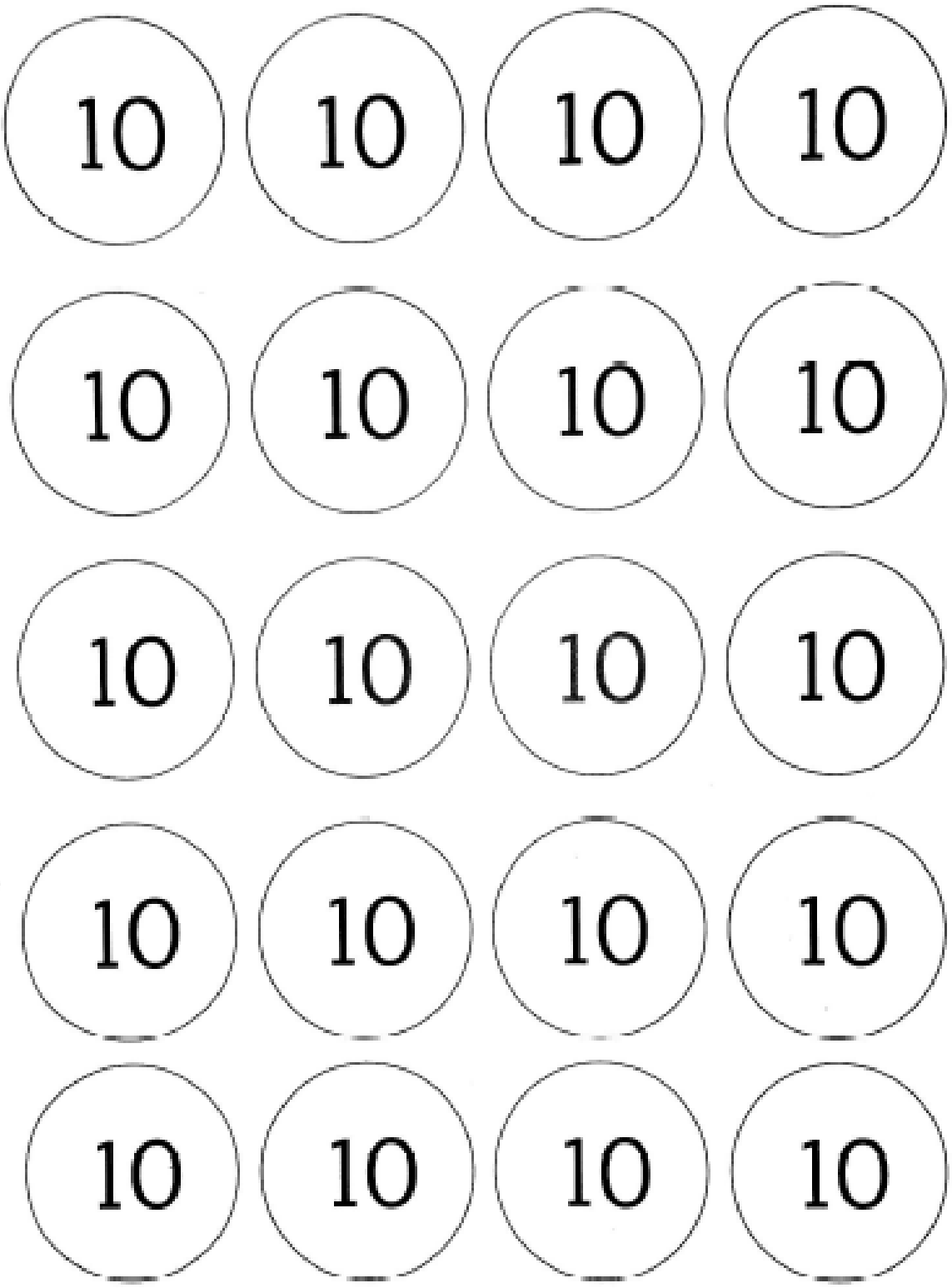
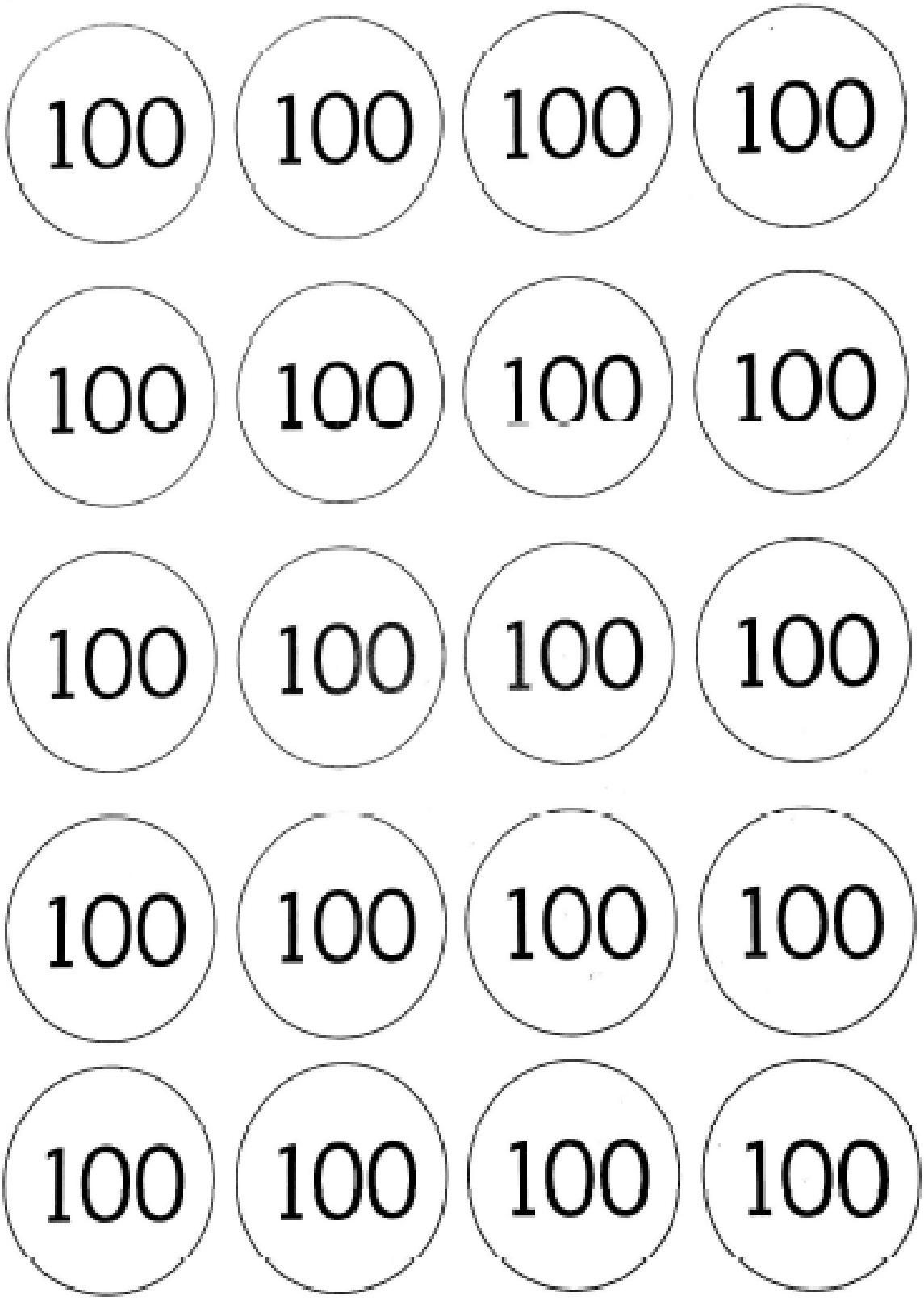
PLACE VALUE DISKS



PLACE VALUE DISKS



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